1 Introduction

Topology is the study of shapes and spaces. Specifically, it's the study of properties that don't change when shapes are stretched, such as the number of holes or twists in a figure. Today we will be exploring some unique ways we can apply topology to Mobius Strips and geometric nets.

2 The Mobius Strip

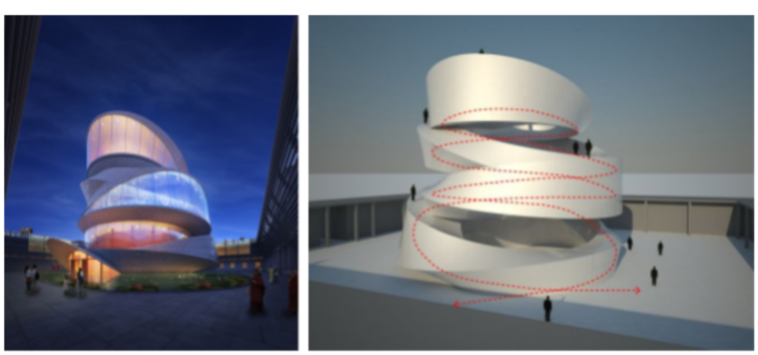
The discovery of the Mobius strip, or Mobius band, is often credited to the German mathematician and astronomer August Ferdinand Mobius, but the intriguing loop was actually described four years earlier by Johann Benedict Listing. The strip is an interesting mathematical figure because it has only one surface, so an ant could crawl along the length of the loop and return to its original starting point, having traversed both "sides" of the strip without ever crossing an edge. Now, we'll start by actually constructing our very own Mobius Strips!

1. Take the 2 ends of the strip, twist it once, and tape the 2 sides together. How many sides does the Mobius Strip have? Hint: try tracing the side of the Mobius Strip with your pencil to see where you end up.
2. What do you predict you will get after you cut the Mobius Strip in half?
3. Now, cut the Mobius Strip in half. What actually happened? Discuss why this might have happened.
4. Now, instead of cutting the Mobius Strip in half, try cutting it into thirds. What happened this time?
5. Now, instead of making a mobius strip with 1 twist before taping it, twist it twice and then tape it. Then, cut it in half. What happened this time?

3 The Mobius Cross

Now, we'll continue our experimentation with Mobius Strips by making Mobius Crosses. For each of the following cases, discuss what happened, and why you think that may have happened.

1. Take two normal loops, and form a cross (with no twists), so it looks like a chain-link. Then, cut each loop in half, as we did before.
2. Now make the same cross, but this time make one ring normal and the other ring twisted. Cut each loop in half.
3. Now make a final cross, but with both rings twisted. Then, cut each loop in half, just as before.



Mobius Strip Temple in Taichang, China

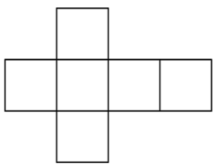
4 Klein Bottles

Klein Bottles are a discovery of German mathematician Felix Klein and share similar characteristics to the Mobius Strip. He had this idea that if you take a cylinder, turn it around, bring it through itself, and have the end welded to the base, then you can create something with one side and no edges. However, these bottles exist only in four dimensional space (4D). As with the Mobius Strip, if an ant were to crawl onto a Klein Bottle, it would be able to travel in and out of the bottle without ever crossing an edge.

5 Nets

3-dimensional objects are not always easy to visualize. That's why we can use nets to represent them, in a 2-dimensional way. A **net** is a pattern you can cut out and fold to make a model of a solid shape. All convex 3-dimensional objects have their own nets.

1. Below is an example of a net of a cube. How many different nets does a cube have? (Hint: draw out all the possibilities)

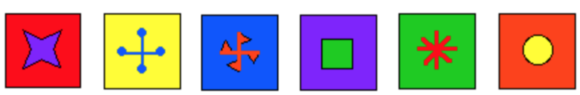


2. Can you create a net for a pyramid? A cone? A cylinder? Draw these on another sheet of paper.

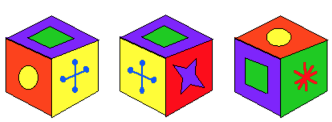
3. How many different nets does a triangular pyramid (tetrahedron) have?

4. Challenge: Suppose you have a cube of side length 3. You cut off one inch from each corner, leaving behind a new figure. How many sides does this new figure have? Can you think of a name for it?

5. Here are the six faces of a cube. They are in no particular order.



Here are three views of the cube, from different angles.



Can you figure out where the faces are in relation to each other and map them on this net of the cube?