1 Order Matters

Permutations are the arrangement of the members of a set. Examples of this include the lunch line and the order of the rides at Dinseyland that you want to ride on. On the other hand, combinations are selections of unordered members of a set. Examples of this would include your English class, the coins in your wallet, and everyone in this room. The difference is that order matters in the case of permutations and order does not matter in the case of combinations. Identify the following groups as permutations, or combinations.

1. The set of notes in Twinkle Twinkle Little Star.
2. Basketball players on a court
3. A locker combination
4. The winning lottery ticket.
5. A group of 3 lecturers from your **fantastic** group of lecturers

2 Permutation Calculations

How many 2 letter words exist? When solving this problem, one would want to break the word down into two letters. If we were to place some letter in the beginning of the word, there would be 26 choices for the second letter. Since there are 26 possible first letters, the answer would be $26 \times 26 = 676$. When solving these kinds of problems, think about how many choices you have for each part to arrive at your final answer. Use the problems below to practice

1. How many ways can someone rearrange the letters in the word CAT?
2. πPhones generally have passwords that consist of 4 numbers. How many πPhone passwords exist?
3. All California license plates follow the format of $OXXXOOO$ where every O represents a number and every X represents a letter. How many license plates have only one letter and one number used?
4. How many ways can someone rearrange the letters in the word SPOON? **BE CAREFUL!!**

3 Combination Calculations

How many groups can be made out of a certain number of people. If I have Jeffery, Rohith, and Clara in a group, is that any different than having Rohith, Clara, and Jeffery in a group? **No**, because order does not matter in this scenario. So how do we calculate the number of combinations? Think about the above situation. If order mattered, you would have 6 different ways to count Rohith, Clara, and Jeffery in a group. However, since order does not matter you should only be counting 1 group. To account for this, you can divide the number of permutations by 6. This same logic can be applied in other scenarios. Use the problems below to practice.

1. In a class of 6, how many ways can you pick one group of 4 students?
2. Mr. Lomas is picking out **HARD** problems for his next algebra test. He wants to pick 3 of his 10 favorite multiple choice questions and 9 of his 10 favorite short answer question. How many different 12 question tests can be made?

4 Stars and Bars

Now that you know how to calculate even groups, an issue arises. What happens when one has to create groups that are not even? How many ways are there to create 3 groups out of 7 pieces of candy. We denote the students as stars as shown below

$$\star\star\star\star\star\star\star$$

We know that we need to split them into groups, so we place bars to denote dividers as shown below.

$$\star|\star\star\star|\star\star\star$$

(**Note** We use two bars because two bars creates 3 groups which is what we want)
The above configuration represents groups of 1 student, 3 students, and 3 students respectively. We want to find the number of ways we can rearrange the stars and bars. After we set up the stars, we see that there are 6 positions for the first bar to be placed in (spaces between stars). Then there are 5 positions left for the second bar to be placed. But don't forget, we are overcounting by a factor of 2. This means that our final answer is $\frac{6*5}{2} = 15$ ways. Use the problems below to practice.

1. Consider the above situation. How many ways can someone create at most 3 groups out of 7 pieces of candy?

2. How many different ways can Billy Bob pick the 5 positive numbers (a, b, c, d, e) such that $a+b+c+d+e = 10$?

5 Challenge Problems

1. How many ways can I make k groups out of n pieces of candy if $k \leq n$?

2. King Arthur had 25 Knights of the Round Table. In fact, he was wondering how many ways he can seat his knights around the round table. Can you help him out if rotations and reflections are not considered distinct? (**Note**: Leave your answer unsimplified)

3. Define a falling number to be a number whose digits are in strictly decreasing order from left to right. For example, 321 and 9431 are falling numbers while 221 and 132 are not. How many 5 digit falling numbers are there?

4. Suhas is currently at Rohith's house, which is at the origin, playing Super Smash Bros Brawl. Suhas wants to return to his own house which is at the point $(7, 4)$. Suhas also wants to stop to get boba tea which is at the point $(4, 1)$. How many different paths can he take to go home, if he can only move up and to the right?

5. The π piper is at the origin of an xy-plane. He wants to get to the point $(2, 2)$. The piper can only move up, down, left, and right by exactly 1 unit with every move. How many ways can the π piper get to his destination if he wants to get there in at most 8 moves?

6. Define an up-down sequence to be a sequence where the any number is exactly one away from the previous number in the sequence. In other words, $|a_{n+1} - a_n| = 1$. Examples of this sequence would be $1, 0, 1, 1, 2, 3, 4, 3$ and sequences that are invalid are $1, 1, 1, 1, 3, 2$. How many up-down sequences of length 15 end in a 5?

7. Refer to the up-down sequences described above. How many up-down sequences of length 30 end in a 5?