## 1 What is Probability?

Many events can't be predicted with absolute certainty. When such is the case, we can at most only say that something is probable. That's when we use the concept of probability! Probability is the chance that something will happen - how likely some event will occur.

Probability is generally regarded as a system of events and outcomes. Most of the time, probability is simply $\frac{\text { specific event(s) }}{\text { all possible outcomes }}$. For example, if we want to know the probability of flipping heads on a coin, we have only one event (heads) out of two outcomes (heads \& tails). Then, our probability is one out of two or $1 / 2$.

## 2 Coin Flipping

### 2.1 Sequences

Imagine we are looking at two sequences of heads and tails. For one of them, I used a random number generator to create this sequence of heads and tails, with an equal probability of the coin being heads or tails. The other sequence, I simply wrote out randomly.

1. H, H, T, T, T, H, H, H, T, T, T, T, H, T, H, T, H, H, H, T, H, H, H, H, T, T, H, T, T, H, H, T, H, T, H, T, T, T, H, H, H, T, H, T, T, T, T, T, T, H, T, T, T, T, H, H, H, T, H, H, T, T, H, H, H, H, T, H, H, T, H, H, H, H, H, T, H, T, H, H, H, T, H, T, H, H, H, T, H, T, H, T, H, T, H, H, H, H, H, T (56 heads, 44 tails)
2. H, H, T, H, H, H, T, T, H, T, T, H, T, H, T, H, H, T, H, T, T, H, T, H, H, T, H, T, T, H, H, H, T, H, H, T, T, H, T, H, T, T, H, H, T, H, T, H, T, T, H, T, T, H, T, H, H, H, T, H, T, T, H, H, T, H, H, T, H, T, H, T, T, H, H, T, H, H, T, T, H, H, T, T, H, H, T, H, T, T, H, H, T, T, H, H, T, H, H, H (54 heads, 46 tails)

Count the number of HHTs and HHHs in each sequence:

Which one is fake (the one I made up), and which one is real (number generator)? Why?

### 2.2 Questions

1. Suppose I flip a coin 100 times. Which is more likely: landing on heads exactly 70 times or landing on heads 71-100 times?
2. Suppose a PMC Lecturer comes up to you and says, "I have a revolutionary coin that comes up tails exactly $51 \%$ of the time and heads the other $49 \%$ of the time. In fact, I even tried it out myself! I flipped the coin 1000 times! It came up heads 511 times.

Is this evidence convincing? If not, how many times would he/she need to flip the coin in order to test out his/her hypothesis that the coin comes up heads $51 \%$ of the time?

## 3 A Game of Dice

Imagine this as a game: your friend rolls two dice and finds the sum of the numbers. You need to predict what sum your friend will get. If you predict correctly, you win the game! What number should you choose so that you win as often as possible?

1. Construct the addition table for numbers from 1 to 6 . What is the smallest sum you can get? What is the largest sum?

| + | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |

2. Amber thinks that all the sums are equally likely. Is she correct? Why or why not?
3. What sum are we most likely to get as the sum of the two numbers on the dice? What sums are we least likely to get as the sum of the two numbers on the dice?

## 4 Teacher's Pick

Now imagine that you are in class, and the teacher asks a question. Since you are such a good student, you raise your hand to answer it! Also, if you get picked and say the right answer, the teacher will like you more.

1. Your class has 30 people in it. Fortunately, not everyone is a good student like you are, so only 15 people (including you) raise their hands. What is the probability you will get picked?
2. However, some people have raised their hands before and answered the teacher's question correctly; they have become favorites. Therefore, the teacher is twice as likely to call on them. What are the chances that you will be called on if 5 people (which don't include you) are favored by the teacher?
3. Surprise! You have a project to present today! However, you aren't quite done. Luckily, there is only enough time to go through five presentations. 10 people have raised their hands to go, and five of them are favorites. The teacher will pick someone who raised their hand 3 out of 10 times, and is twice as likely to pick the favorites. What is the probability you will get picked if you raise your hand the whole time(and you are not a favorite)? What is the probability that you will get chosen if you do not raise your hand the whole time? How do you maximize your chances of not getting picked (so you have another night to work on your project)?
