## 1 Patterns with Exponents

A base is a number that you multiply. An exponent tells us how many times to multiply by that number. For example, in  $2^3$ , 2 is the base and 3 is the exponent, and it means  $2^*2^*2$ , since we multiply three 2's together. When we say "2 to the power of 3," we mean  $2^3$ .

- 1. What is the last digit in  $9^{123}$ ? (This means  $9 \times 9 \times \dots 9 \times 9$  where there are 123 9's.) 9
- 2. What is the last digit in  $5^{11}$ ? 5
- 3. What is the last digit in  $7^{2013}$ ? 7
- 4. What is the **second to last** digit in  $6^{17}$ ? 3
- 5. What is the **last two** digits in  $7^{999}$ ? 43
- 6. Find the sum of
  - a) the first 10 multiples of 2 110
  - b) the first 10 multiples of 3 165
  - c) the first 10 multiples of 5 275
- 7. **Bonus:** What is the last two digits in  $41^{2789}$ ?

## 2 Other Questions

- 1. Find the sum of the numbers from 1 to 19. Hint: Don't actually add them all...
- 2. Now, find the sum of the numbers from 1 to 99.
- 3. For each set of pictures, what number will you write in the place of X to continue the pattern?





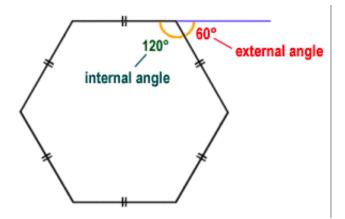




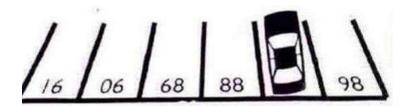




4. As the below picture shows, polygons have both internal and external angles. In a regular polygon, all the sides have the same length and all the angles have the same measure; for example, a triangle would have external angles of 120 degrees, a square 90 degrees, and a pentagon 72 degrees. With this in mind, take a look at the following pattern, which contains the external angles of various polygons: 120, 90, 72, 60, x, 45, 40, 36, y, 30, z, ... What are the values of x, y, and z? (Hint: Fractions might be easier to work with here!) Respectively, 360/7, 360/11, 360/13



- 5. What are the next six numbers in this pattern? 0, 4, 2, 3, 4, 4, 6, 3, 8, 5, 10, 3, 12, 6, 14, 8, ... The next 6 numbers are 16, 7, 18, 8, 20, 6. (even number followed by number of letters in even number)
- 6. What is the sum of all the odd numbers from 37 to 135, inclusive? ("Inclusive" means that you count 37 and 135 as well.) 4300
- 7. What is the number under the car?



## 3 References

Berkeley Math Circle