PMC Infinity: Spooky Edition

Pleasanton Math Circle

10/18/18

1 A real number or a concept?

You have probably heard of infinity sometime in your life. Maybe it was in your math class or maybe it was from the new movie Infinity War. Either way, infinity is an important, yet intricate, **concept** to grasp.

2 A bigger number

Think of the largest number that you know. Guess What... there exists another number that is larger than the number you thought of, and another number larger than that one, and so on. Mathematicians from a long time ago decided to encompass this idea of the largest number, and they proposed the concept of infinity.

3 Practice

Let $x = \infty$, $A = (2x)^x$, and $B = (3x)^x$

1. What is A?

2. What is B?

3. What is $A + B$?

4. What is $A - B$?

5. What is $A * B$?

6. What is $A \div B$?

4   Escaping from Dracula

You thought it would be a great idea to enter the dark mansion after your best friend double dog dared you to. You did not expect to come face to face with the king of vampires himself, Dracula. Instead of engaging in a long and nonmathematical chase with him, Dracula makes you an offer.

He tells you to stand where you are while he mathematically comes towards you. Dracula will cover half the length between you and him, and then stop. Then he will cover half of the new length between you and him, and then stop. Dracula will keep continuing this process. If you agree to Dracula's terms, will you survive?

5   Trick or Treater Dilemma

You are an avid trick or treater so you plan on trick or treating at an infinite number of houses on a certain block. These house are conveniently numbered $1, 2, 3 \ldots$. Unfortunately, there is someone already trick or treating at every house on the block. This is a problem!!! You can't get candy from a house if someone is already getting candy. However, you are a smart cookie. you wait until everyone is done trick or treating at the house they are at and you wait for them to move on to the next house. Is there space for you to begin your trick or treating adventure?

Note: This concept is better known as the hotel problem.

6   Cantor's Haunted House

You finally arrived at Cantor's house after a long and tiring Halloween Day. You rang the doorbell, but instead of receiving candy, Cantor gave you a math problem (What could be scarier on Halloween). To make matters worse, he gave you a proof-based problem. He threatened that if you could not solve his question, you would be forced to use $\pi = 3$ for the rest of your life (It is worth noting that Cantor has no friends). Is there a way to count the number of strings of infinite length composed of 0s and 1s? Some examples of infinite strings of this kind include $11111111 \ldots, 00000000 \ldots, 10101010 \ldots$ and so on and so forth.