PMC Game Theory

Pleasanton Math Circle

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1 What is Game Theory?

When we say Game Theory, it is not just limited to video games or board games. In fact, the formal definition of game theory is

The branch of mathematics concerned with the analysis of strategies for dealing with competitive situations where the outcome of a participant's choice of action depends critically on the actions of other participants.

2 Nim

The basic Nim begins with two players and several heaps, each containing several objects. Occasionally, heaps are also called piles, and the objects are called stones.

Each player, in turn, must take at least one stone, but they may take more than one stone as long as they all come from the same pile. It's allowed to make a pile empty, effectively removing the pile out of the game. When a player is unable to move, the game ends. Naturally, as long as there is a stone, either player can take that stone, and thus can move. So the ending condition can be rephrased, where the game ends if there is no stone left. In normal Nim, the loser is the player unable to move.

3 Bases

Although this may seem unrelated to the topic of the day at first, it may pose some relevance later on... Everyone counts in base 10. What does this mean? Well when we represent the number 6803, we are using 6 thousands, 8 hundreds, 0 tens, and 3 ones (Hopefully this sounds familiar). Interestingly enough, we are always adding multiples of powers of ten. We are adding $6 \times 10^3 + 8 \times 10^2 + 0 \times 10^1 + 3 \times 10^0$. Counting in base n will work in the exact same fashion except with n replaced with all the tens.

Practice Problems:

1. Convert 32_{10} into base 2. Note: the subscript is the base that the number is represented in.

2. Convert 101_2 into base 4

3. Convert $10101010101...1010101_2$ into base 4

4. Find another way to convert from base 10 to any other base using division instead of subtractions

4   Guess $\frac{2}{3}$ of the average

Here is a much simpler (yet just as challenging) game. The goal is simple. You and another person both think of a number and write it down so that the other person cannot see your number. After you both are done, you compare your two numbers, and the guess that is closer to 2/3 of the average of both numbers will win.

Example: If one person guesses 5, while the other person guesses 7, the person with a guess of 5 will win. Since the average of 5 and 7 is 6, $\frac{2}{3} * 6 = 4$. The guess closer to 4 wins!!!!

Seems simple enough right? What is the ideal strategy to win?

5   Robot Ricochet

Finally we have a fun game called Robot Ricochet. The goal is simple, get the right color robot into the right colored square in the shortest number of moves possible. The definition of a move is that

1. A robot (colored circle) must move as far as possible in one of the four direction in one move

2. The robot stops when it hits either a wall or another robot

How many moves does it take you?