1 What are my Chances?

What is the likelihood of flipping heads when flipping a coin once? Most of you would answer 1/2 in a small amount of time. But what is really going on here? You have just calculated the probability of flipping heads!

Probability is nothing but the number of favorable outcomes over the total number of outcomes. In the above case, we see that the favorable outcome was heads, which is one outcome, and the total number of outcomes was 2. Therefore, the answer is 1/2.

Try it by yourself?

1. What is the probability that you roll a 1 when you roll a die?
2. There are 11 Soccer players on a court. What is the probability Lionel is the goalie?
3. The winning lottery has 5 random digits. What is the probability that you randomly select the correct one?
4. 6 ice creams must be chosen from among four flavors: Vanilla, Chocolate, Strawberry, and Orange. What is the probability that I choose 1 vanilla, 1 chocolate, 1 strawberry, and 3 oranges?

2 Independence

I'm sure you have heard your parents tell you a countless number of times that you need to become independent, so you probably have a good idea of what the word means. In probability, it is very similar. When two events are independent of each other, that means that the outcome of one event does not affect the outcome of another. For example, tossing a coin and rolling a dice are two independent events.

1. Can you think of two events that are **NOT** independent?

3 Mutually Exclusive

When two events are mutually exclusive, it means that the chances of both events occurring is 0. For example, rolling a 2 or a 3 on a single roll are mutually exclusive. I cannot roll both a 2 or 3 on a single dice at the same time.

1. What are other events that are mutually exclusive?

4 And vs Or

One of the most common misconceptions is that two independent events are always mutually exclusive. Let us try to come up with some counterexamples. (Note: Some of the following scenarios may not be possible)

1. What are two events that are mutually exclusive AND independent?
2. What are two events that are mutually exclusive, but NOT independent?
3. What are two events that are NOT mutually exclusive, but are independent?
4. What are two events that are neither mutually exclusive NOR independent?

Now let us try calculating some of these probabilities. A lot of times, the wording helps determine whether to add or subtract. If I ask for the probability of event A **AND** event B happening, I would find this by multiplying $P(A \cap B) = P(A) * P(B)$ because I need both event A and event B to happen. You might be thinking that if the question asks for the probability of event A **OR** event B happening, you would simply add the two probabilities together, but this is NOT the case. Consider the following problem

1. $\frac{3}{4}$ of the students at Harvest Park have Mr. Lomas as their math teacher. $\frac{3}{4}$ of the students at Harvest Park have Mr. Samol as their htam teacher.

If we were to simply add the probabilities in this case, we would believe that $\frac{3}{2}$ of the students at Harvest Park have either Mr. Lomas or Mr. Samol as their teacher, but this is clearly impossible. We're forgetting that some people have both Mr. Lomas and Mr. Samol as their teacher. We actually need to subtract out the people who have both Mr. Lomas and Mr. Samol as their teacher, since we already counted it once. This can also be shown with a Venn Diagram. The general formula ends up being $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

5 How to Count

Counting is one of the most basic concepts in math. However, when it comes to probability, there are 2 things one must count. The total number of outcomes and the number of favorable outcomes. Below are some ways that you can do this.

6 Cases and Complementary Counting

Let us say I slightly modified question 1 in the previous section and said: In a 4 digit locker combination, what is the probability that I have at least 1 six? You could solve this problem like this: There are 4 choose 1 ways to have 1 six, multiplied by 9^3, and there are 4 choose 2 ways to have 2 sixes, multiplied by 9^2, and there are 4 choose 3 ways to have 3 sixes, multiplied by 9^1 and there is one way to have all sixes. Adding them up and dividing by 10^4, we get 3439/10000

We could also find the probability of there being no sixes at all and then subtracting that from the total number of combinations to find the number of ways there is at least one six. There is a 9/10 probability a certain number is not a six, so there is a $(9/10)^4$ probability that there is no six at all. Subtracting from 1, we get 3439/10000.

1. Paul flips a fair coin eight times. In how many ways can he flip at least two heads?

2. How many squares can be formed using the vertices of a 4x4 lattice grid?

7 Binomial Probability

How many of you guessed on the AMC 8 or AMC 10? How many of you have had that multiple choice test where you didn't know anything? Binomial probability is a probability formula for Bernoulii trials. A Bernoulii trial is basically performing an action over and over again that has only two outcomes: success or failure. When you guess a question, there are only two possibilities: right or wrong.

With this, let us say I guess the last 5 questions on AMC 10. What is the probability of getting exactly 3 of them right?

The formula for k successes in n trials is $\binom{n}{k}(p^k)(q^n - k)$, where p is the probability of success and q is the probability of failure Using this, we get that the probability is $10 * (1/125) * (16/25)$, or 32/625

8 Problems

1. Each of the 6 faces of a cube is painted either black or white with equal probability. What is the probability that no two black faces share an edge?

2. A number m is randomly selected from the set $\{11, 13, 15, 17, 19\}$, and a number n is randomly selected from $\{1999, 2000, 2001, \ldots, 2018\}$. What is the probability that m^n has a units digit of 1?

3. In a high school with 500 students, 40% of the seniors play a musical instrument, while 30% of the non-seniors do not play a musical instrument. In all, 46.8% of the students do not play a musical instrument. How many non-seniors play a musical instrument? (AMC 10B 2019 #3)

4. A red ball and a green ball are randomly and independently tossed into 3 bins numbered with positive integers so that for each ball, the probability that it is tossed into bin k is $\frac{k}{6}$ for $k = 1, 2, 3$ What is the probability that the red ball is tossed into a higher-numbered bin than the green ball? (Spinoff of AMC 10B 2019 #17)

5. Amelia has a coin that lands heads with probability $\frac{1}{3}$, and Blaine has a coin that lands on heads with probability $\frac{2}{5}$. Amelia and Blaine alternately toss their coins until someone gets a head; the first one to get a head wins. All coin tosses are independent. Amelia goes first. What is the probability that Amelia wins? (AMC 10A 2017 #18)