Venn Diagrams

Pleasanton Math Circle: Middle School

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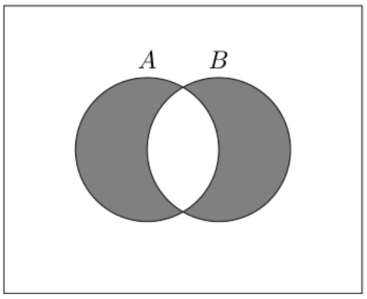
September 26, 2024

§1 What is a Venn Diagram?

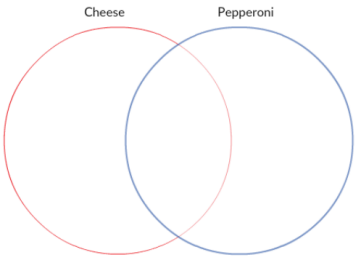
When we are solving a problem with addition and subtraction, it is sometimes helpful to use a picture, or a diagram, to solve that problem:



The picture above is called a Venn diagram. In the diagram, we draw Circle A for one group and Circle B for another group. In the circle overlap, we have a group that is in both A and B.

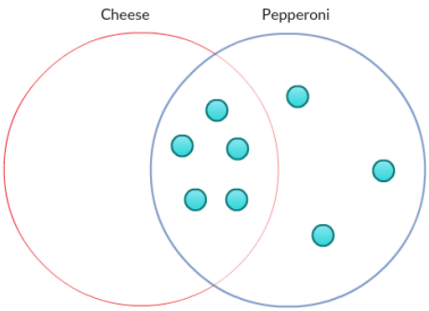
§2 Draw a diagram and solve

At Mathnerd Elementary School, there are 12 kids in one class. All of the kids are eating at least one slice of pizza. They are offered cheese or pepperoni pizza. 8 of the kids ate pepperoni pizza and 5 of the kids ate both cheese and pepperoni pizza. How many kids ate cheese pizza?



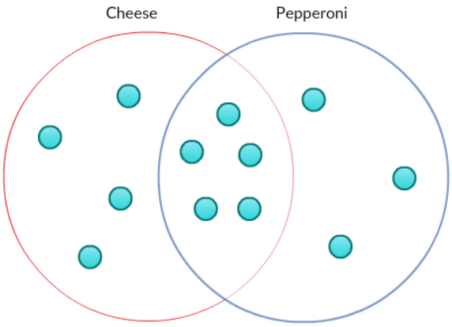
Alright! So let's try using the Venn diagram. We'll place points in the circles to represent the kids - one point per kid. Let's go!

We first place five points in the intersection space inside of both circles, because there are five kids that ate both pizzas. Now, since there are 8 kids eating pepperoni pizza, and 5 points are already inside the intersection, there must be three more points inside the pepperoni circle which aren't in the cheese circle!



Since there are 12 kids, there must be 4 left which are inside the cheese circle but not the pepperoni circle.

We've placed all of our kids into the diagram. Now we can just read off the answer to our original problems.



There are 9 points inside the cheese circle on the left. So 9 kids ate cheese pizza!

Quick Question: Why did we start with the 5 kids who ate both pizzas rather than with the *12 total kids* or the *8 people eating pepperoni*?

§3 Do it yourself

1. At Pleasanton Library, there are 49 visitors who checked out books. The library has two sections: Fiction (Set A) and Non-Fiction (Set B). There are 27 visitors who checked out Fiction books, and 29 visitors who checked out Non-Fiction books. Some visitors checked out both types of books. How many visitors checked out both Fiction and Non-Fiction books?

2. In a school of 220 students, every student plays basketball, soccer, or both. If 195 students play basketball and 60 students play soccer, how many of the basketball players do not play soccer?

3. In a summer camp of 41 kids, 24 swim, 16 hike, and 10 play volleyball. The number of kids who swim and play volleyball is twice the number of kids who swim and hike, and no kids participate in both hiking and volleyball. Determine: a) How many kids swim and play volleyball. b) How many kids swim and hike.

4. In the imaginary Pleasanton zoo, there are 30 leopards. These leopards have orange spots or purple spots (or maybe both!). 20 leopards have orange spots, while 13 leopards have both orange and purple spots. How many leopards only have purple spots?

5. There's a candy store that sells chocolate bars and lollipops. A customer can only buy one per candy. On one lovely afternoon, there were 66 chocolate bars and 59 lollipops sold. 19

customers bought both a chocolate bar and a lollipop. How many customers came in that afternoon?

6. A carnival has 40 clowns. 25 clowns tell jokes and 31 clowns tell riddles. 5 clowns don't tell anything. How many clowns can tell both jokes and riddles? (*WARNING: This question may require some algebra!*)

§4 Challenge Problems!!!

1. In a raffle drawing, ticket numbers range from 1 to 50. How many ticket numbers are divisible by 3? Divisible by 7? How many ticket numbers are divisible by 3 but not by 7? How many are divisible by 7 but not by 3? Try representing this with a Venn diagram. How many ticket numbers are neither divisible by 3 nor by 7?

2. At Techno High, every student takes either programming or robotics, or both. Let x be the number of students in programming, y the number of students in robotics, and z the number of students in both. How many students are at Techno High in terms of x, y, and z?

3. At Harvest Park, there are 200 students. Out of these, 100 are enrolled in Math 8, 80 are in Algebra I, and 50 are taking Geometry. Among the math students, 20 are taking only Math 8 and Algebra I, 15 are taking only Math 8 and Geometry, and 10 are taking only Algebra I and Geometry. Additionally, 5 students are enrolled in all three math classes. How many students (the non-cool kids) at Harvest Park are not enrolled in any math class?

4. *2015 AMC 8 Problems/Problem 15* - At Euler Middle School, 198 students voted on two issues in a school referendum with the following results: 149 voted in favor of the first issue and 119 voted in favor of the second issue. If there were exactly 29 students who voted against both issues, how many students voted in favor of both issues?