1. Introduction

Sometimes, math can show us surprising results that seem to defy logic, like finding a way to "solve" a problem that leads to a contradictory answer. These results are called paradoxes, and they are like tricky puzzles that challenge our understanding!

2. Can you find the problem?

1. $-20 = -20$
   $25 - 45 = 16 - 36$
   $5^2 - 5 * 9 = 4^2 - 4 * 9$
   $5^2 - 5 * 9 + 81/4 = 4^2 - 4 * 9 + 81/4$
   $(5 - 9/2)^2 = (4 - 9/2)^2$
   $5 - 9/2 = 4 - 9/2$
   $5 = 4$
2. $1 = \sqrt{1} = \sqrt{(-1) * (-1)} = \sqrt{-1} * \sqrt{-1} = i * i = -1$

3. Which statement(s) is correct?

1. If all alligators are ferocious creatures and some creepy crawlers are alligators, which statement(s) must be true?

I. All alligators are creepy crawlers.

II. Some ferocious creatures are creepy crawlers.

III. Some alligators are not creepy crawlers.

(Source: 2000 AMC 10)

2. Ms.Carroll promised that anyone who got all the multiple choice questions right on the upcoming exam would receive an A on the exam. Which one of these statements necessarily follows logically?
(A) If Lewis did not receive an A, then he got all of the multiple choice questions wrong.
(B) If Lewis did not receive an A, then he got at least one of the multiple choice questions wrong.
(C) If Lewis got at least one of the multiple choice questions wrong, then he did not receive an A.
(D) If Lewis received an A, then he got all of the multiple choice questions right.
(E) If Lewis received an A, then he got at least one of the multiple choice questions right.

(Source: 2017 AMC 10)

3. Malcolm wants to visit Isabella after school today and knows the street where she lives but doesn't know her house number. She tells him, "My house number has two digits, and exactly three of the following four statements about it are true."
(1) It is prime.
(2) It is even.
(3) It is divisible by 7.
(4) One of its digits is 9.
This information allows Malcolm to determine Isabella's house number. What is its units digit?
(A) 4 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

(Source: 2017 AMC 8)

4. Challenge Problems

1. let a = b
   Then $a^2 = ab$
   a^2–$b^2 = ab$–b^2
   $(a-b)(a+b) = b(a-b)$
   $a + b = b$ (divide by a-b)
   $b + b = b$ (as a = b)
   $2b = b$
   $2 = 1$
2. $0 = 0 + 0 + 0 + 0 \ldots$
   0 = (1-1) + (1-1) + (1-1) + (1-1) ...
   0 = 1-1+1-1+1...
   0 = 1 + (-1+1) + (-1+1) + ...
   0 = 1
3. Let a = b = -1
   $a^2 = b^2$
   $2a^2 = 2b^2$
   $a^2 = 2b^2 - a^2$
   a = $\sqrt{(2b^2 - a^2)}$
   a = $\sqrt{(2(-1)^2 - (-1)^2)}$
   a = $\sqrt{(1)}$
   - 1 = 1
4. HARD WARNING
   Given a $\triangle ABC$, prove that AB = AC
   1. Draw a line bisecting $\angle A$
   2. Draw the perpendicular bisector of segment BC, which bisects BC at D.
   3. Let these 2 lines meet at point O.
   4. Draw line OR perpendicular to AB, line OQ perpendicular to AC
   5. Draw lines OB and OC
   6. By AAS, $\triangle RAO \cong \triangle QAO (m\angle ORA = m\angle OQA = 90; \angle RAO \cong \angle QAO; AO = AO)$
   7. By HL, $\triangle ROB \cong \triangle QOC (m\angle BRO = m\angle CQO = 90; BO = OC; RO = OQ)$
   8. Thus, $AR \cong AQ, RB \cong QC$ and $AB = AR + RB = AQ + QC = AC$