1 Introduction

1.1 Definition:

Definition 1. Hyperbolic geometry: The type of geometry you get assuming all of Euclid's postulates except his fifth, which is reversed

1.1.1 Euclid's Postulates:

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center.
4. All right angles are congruent.
5. If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. This postulate is equivalent to what is known as the parallel postulate.

2 Properties:

1. Rectangles do not exist in hyperbolic geometry
2. Angles in a triangle add up to less than 180
3. Similarly, the angles in a quadrilateral add up to less than 360
4. Can be generalized for more polygons
5. Circles have a circumference less than 2π
6. If two triangles are similar, then they are congruent

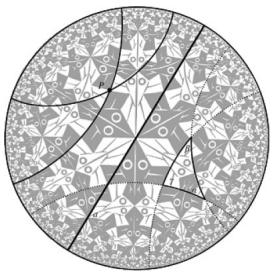
3 Examples:

1. "Triangle in a hyperbolic paraboloid"-aka saddle shape

There are three ways you can visualise a hyperbolic plane on a flat surface.

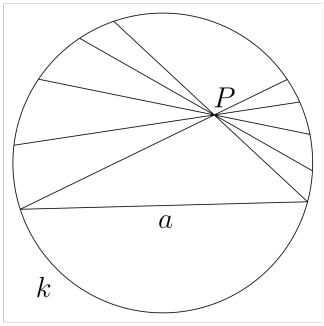
1. POINCARE DISK

   On a Poincare disc, the straight lines look like lines that you draw on a sphere. This model preserves the angles of figures. Also, the circle represents an infinite line that things never reach.



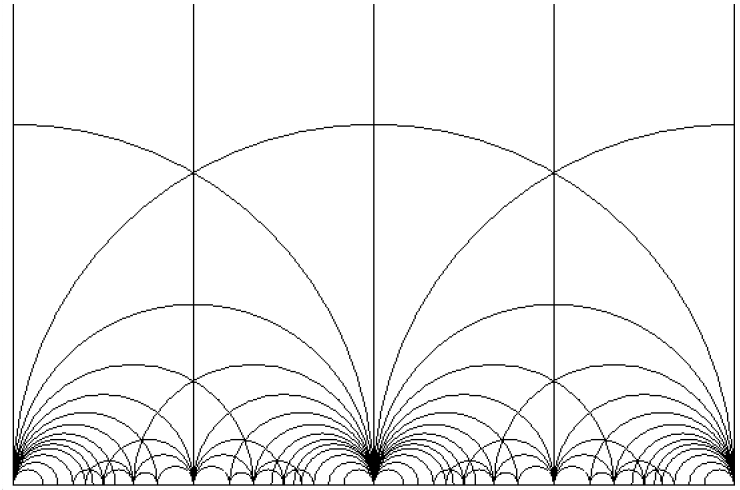
2. KLEIN DISK

   On a Klein disk, all the lines that are meant to be straight on the hyperbolic plane, are straight as well. Triangles on a hyperbolic plane look like regular Euclidean triangles. Though this means that the angles are not preserved in this model. The circle represents the infinite line that things can never reach.



3. POINCARE HALF PLANE

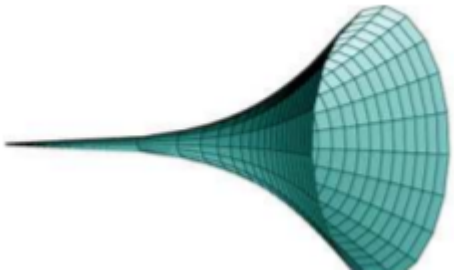
   On a Poincare half plane, straight lines look like half circles or like straight lines in Euclidean geometry, and the line on the bottom represents infinity. This visualisation preserves the angles of figures.



4 Applications

Many cosmologists, or scientists that study the origin and the evolution of the universe, think that hyperbolic planes might describe the shape of our universe. There is a couple of different hyperbolic structures, suggested by scientists.

Multi-connected hyperbolic horn is a shape that has a structure of a cone that infinitely expands from one side and infinitely shrinks on another.



Another possible type of the structure of our universe is called compact hyperbolic, which is an inclosed space that looks like a double torus, or a double donut. We, as observers, will be sitting inside of this torus, watching the universe.