

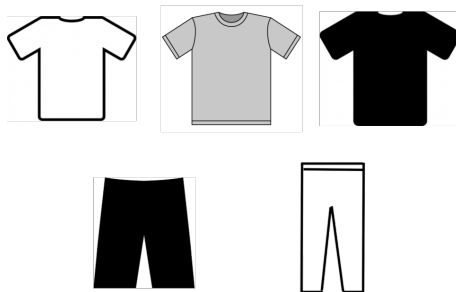
Counting Things

Pleasanton Math Circle: Elementary School

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§1 Counting with Multiplication



Problem 1.1. Asher has three shirts - (a black shirt, a gray shirt, and a white shirt) and two pairs of pants (a black pair and a white pair). How many outfits consisting of one shirt and one pair of pants can he wear to the dance?

Theorem 1.2

We can use multiplication to count a series of independent events.
Independent means that each decision does not depend on the others.

Problem 1.3. Mr Lamos is making a sandwich. He can choose one type of bread from Sourdough, Focaccia, or Rye; one type of cheese from Parmesan, Swiss, or Cheddar; and one type of meat from Ham, Bacon, or Beef. How many different sandwiches can he make?

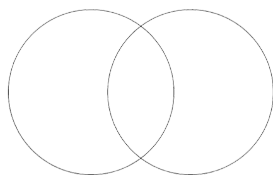
Problem 1.4. Asher is flipping a coin 3 times. How many different sequences of heads or tails can she get?

Problem 1.5. (Challenge) Asher is flipping a coin 6 times. How many ways can she flip exactly 2 heads?

§2 Counting with Addition and Subtraction

Theorem 2.1

Venn Diagrams are useful for counting things in overlapping groups.
When using a Venn Diagram start filling in the diagram with numbers that we know go in a single region of the diagram. Usually, this means that we start in the middle of the diagram.



Problem 2.2. Asher has four friends: Joe, Donald, Mike, and Nancy. Emily has three friends: Joe, Jayden, and Jasper. How many different friends do they have in total? (Hint: Draw a Venn Diagram).

Theorem 2.3

If we have two groups, the total size of the groups is the size of the first group + the size of the second group - the number of items in both groups.

Problem 2.4. Parvest Hark has 94 students, which either take a math class, a writing class, or both. 71 students are taking a math class, and 55 students are taking a writing class. How many students are only taking a math class?

Problem 2.5. Mr. Lamos has 69 students. 40 of them have black hair, 42 of them have a pet, and 24 of them have green eyes. If 15 of them have black hair and green eyes, 20 of them have black hair and a pet, and 13 of them have green eyes and a pet, how many students have black hair, green eyes, and a pet?

§3 Probability

Problem 3.1. Asher rolls a die. What is the probability he gets a 3?

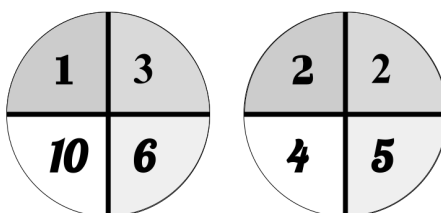
Theorem 3.2

Probability is defined as the number of "successful" outcomes divided by the number of total outcomes. When using the above theorem, it is important that each outcome is equally likely!

Problem 3.3. Emily is rolling two dice. What is the probability that the sum of the two dice is 7?

Problem 3.4. Asher is flipping six coins. What is the probability that exactly one of them is heads?

Problem 3.5. Asher spins each of the two spinners shown below. What is the probability that he gets a 1 on the first spinner and a 2 on the second spinner?



Theorem 3.6

If you have two independent events, the probability that both occur is: the probability that the first occurs \times the probability that the other appears.

Problem 3.7. In Vamador Alley, the probability that it rains tomorrow is 30%, and the probability that it rains the day after is 70%.

What is the probability that it rains both tomorrow and the day after?

What is the probability that the weather tomorrow and the day after is the same?

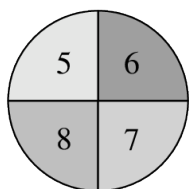
Problem 3.8. Asher is attending a session of the Pasadena Music Club (PMC). The probability that he solves question 1.1 is 50%, the probability that he solves problem 2.2 is $\frac{2}{3}$, and the probability that he solves question 3.8 is $\frac{7}{10}$. What is the probability that he solves problem 1.1 and 2.2, but doesn't solve 3.8?

Problem 3.9. (Challenge) Emily is a doctor, and 5% of the population has the mysterious disease DIVOC-91. If you don't have DIVOC-91, Emily has a $\frac{94}{95}$ chance to diagnose you correctly, and if you do have DIVOC-91, Emily has a 100% chance to diagnose you correctly. If Emily diagnoses you with DIVOC-91, what is the probability that you actually do have DIVOC-91?

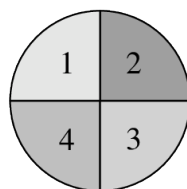
§4 Challenging Problems

Problem 4.1. At Parvest Hark, 198 students voted on two issues with the following results: 149 voted in favor of the first issue and 119 voted in favor of the second issue. If there were exactly 29 students who voted against both issues, how many students voted in favor of both issues?

Problem 4.2. The two spinners shown below are spun. Let the number N equal 10 times the number on Spinner A, added to the number on Spinner B. What is the probability that N is a perfect square number?



Spinner A



Spinner B

Problem 4.3. One member of the saxophone section is going to be first chair. Each member is equally likely to be chosen. The probability that boy is chosen is $\frac{3}{4}$ the probability that a girl is chosen. How many people are in the saxophone section?

Problem 4.4. A bag contains only blue balls and green balls. There are 6 blue balls. If the probability of drawing a blue ball at random from this bag is $\frac{1}{4}$, what is the number of green balls in the bag?