

1 Introduction

Welcome to the second Pleasanton Math Circle meeting of the year! We are so excited to start working with all of you and solving fun, challenging, and thought-provoking math problems together. This week, we are working on familiarizing ourselves with basic statistics concepts. Try the warm-up and ask a teacher if you need help.

2 Warm Up: Mean, Median, and Mode

1. Mean: the average of all of the numbers in a data set.
2. Median: the value that is in the middle of a data set that is sorted in increasing order.
3. Mode: the value(s) that occur the highest number of times.

Here's an example of the mean, median, and mode:

Take this set: 1, 1, 4, 6, 2, 3, 5, 3, 5, 1, 2

The mean of these numbers would be: $(1 + 1 + 4 + 6 + 2 + 3 + 5 + 3 + 5 + 1 + 2)/11 = 33/11 = 3$

To find the median, we put the numbers in order: 1, 1, 1, 2, 2, 3, 3, 4, 5, 5, 6 The middle number is 3, so the median is 3

To find the mode, we could up the number of occurrences of each number. 1 appears 3 times, which is the most, so 1 is the mode

Find the mean, median, and mode of the following datasets:

1	2	5	8	10
6	3	0	9	0

1.
4.4

6.5	2.8	3.6	9.1	9.3
9.7	7.9	10.0	2.8	4.2

2.
6.59

72	98	13	65	74
36	95	97	32	68

3.
65

3 Applying Statistics

In this section, we will do some problems that apply the concepts in the previous section

1. Suppose the following numbers are Ryan's grades on tests this school year:

73, 84, 100, 91, 92, 96, 84

What is your mean, median, and mode test score?

Mean: 89. Median: 91. Mode: 84.

2. In the first four of Homer's five bowling games, he gets scores of 212, 184, 165, and 173. What must Homer bowl in his fifth game in order to make his average score over the five games be 190?
216
3. Mary has five bags of candy. The numbers of pieces in the bags are 6, 8, 12, 14, and 15. What is the average number of pieces per bag? If Mary adds 23 pieces to each bag of candy, what will be the new average? Do you notice anything about this number?
13.75
4. In a list of positive numbers, all have different values. Their sum is 350. Their average is 50. One of the integers is 100. What is the greatest integer that can be in the list?
There are 7 elements since $350/5 = 7$. The lowest possible value is 1, so we can say 5 items in the list are 1. We have the list 1, 1, 1, 1, 1, 100, and x. Since everything adds up to 350, $x = 245$.
5. In science class, the teacher assigns each student a semester grade by finding the median of that student's test scores. There are seven tests each semester. If a student has scored 55, 78, 63, and 91 on the first four tests, what is the highest possible semester grade he or she can earn?
The highest possible median is 91 since the median of a 7-element list is the 4th element. Therefore, for the remaining 3 elements, they need to be ≤ 91 in order to maximize the median.

4 Limits of Statistics

While statistics are very useful, they also have some limits. An example of these limits is below:

You are taking a partner quiz in Mrs. Seyer's class, and since she is nice and actually a good teacher, she allows you to choose your partner! Obviously, you would like to maximize your score on the quiz. You have four available people to choose from. Their names and scores are below:

Anna: 94, 93, 90, 93, 92, 91, 0

Bob: 98, 94, 33, 33, 96, 97, 23

Carol: 89, 88, 86, 88, 87, 84, 85

Doug: 100, 14, 3, 100, 11, 2, 21

Which student has the highest mean score? Highest median score? Highest mode score? Which student would you choose to be your partner?

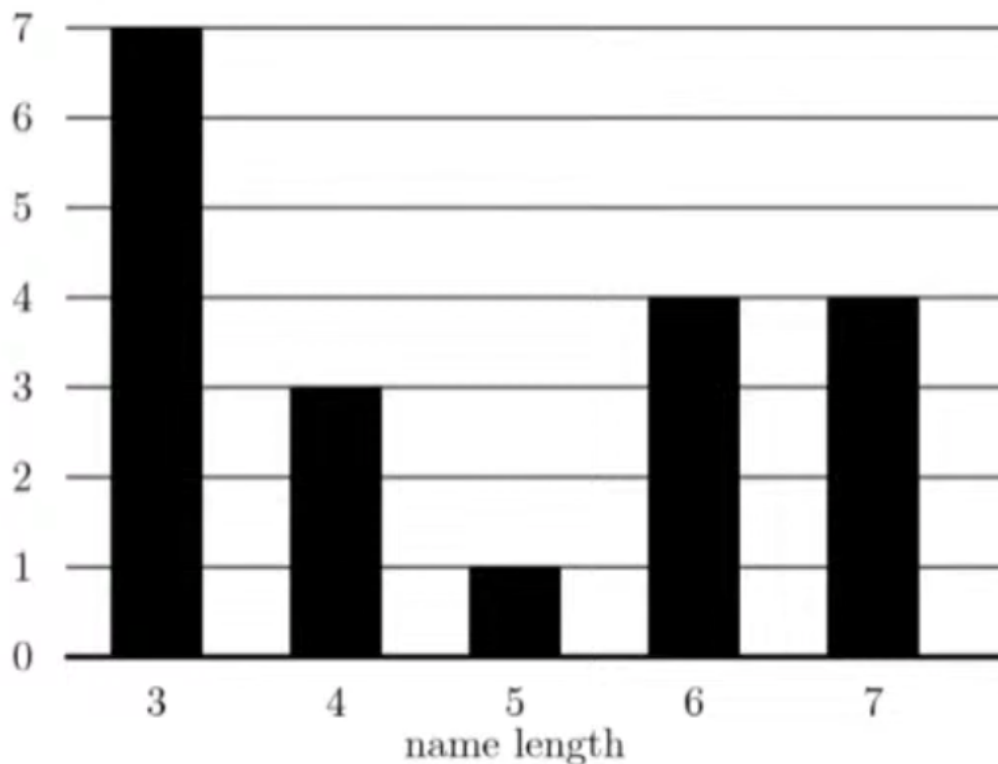
Mean: Carol. Median: Bob. Mode: Doug. However, we would choose either Anna or Carol since they are the most consistent.

5 Challenge Problems

1. What is the smallest possible mean of four distinct positive even 2-digit integers?

$$(10 + 12 + 14 + 16)/4 = 13.$$

2. The following bar graph represents the length in letters of the names of people. What is the mean, median, and mode of name length?



Mean: $(7 \times 3 + 3 \times 4 + 1 \times 5 + 4 \times 6 + 4 \times 7)/(7+3+1+4+4) = 4.74$. Median: 10th element = 4. Mode: 3.

3. When the mean, median, and mode of the dataset 10, 2, 5, 2, 4, 2, x are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real values of x?

Mean: $(x+25)/7$. Median: 2, 4, or x. Mode: 2. Median can't be 2 since the mode is already 2. If the median is 4, then the mean can be either 0, 3, or 6. From this we get $x = -25, -4, \text{ or } 17$. If the median is x, $x = 36/5$. $-24 - 4 + 17 + 7.2 = -3.8$.

4. The mean, median, and unique mode of the positive integers 3, 4, 5, 6, 6, 7 and x are all equal. What is the value of x?

$$(x+31)/7 = 6, x = 11.$$