

## 1 Introduction

Welcome to the first Pleasanton Math Circle meeting of the year! We are so excited to start working with all of you and solving fun, challenging, and thought-provoking math problems together.

For a fun start to the year, we're working on riddles and deductive reasoning today. Deductive reasoning is a process where you come up with specific conclusions based on general ideas that you are given. Try the warm-up and ask a teacher if you need help.

## 2 Warm-Up

1. The rosebush needs sunlight.
2.  $A = C$
3. From first to last: C, A, B, D, E

## 3 Getting a Pet

Zach: Guinea Pig, \$25

Rachel: Parrot, \$100

Hubert: Snake, \$50

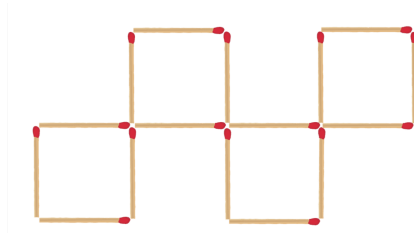
## 4 Riddles!

1. The number of hens has increased from 1.5 to 6 (4 times more) and the number of days has increased from 1.5 to 6 (also 4 times more). The effect is cumulative, therefore the number of eggs that will be laid will be 16 times more (i.e. 24 instead of 1.5). **Answer: 24**
2. Farmer takes Goat across (leaving Wolf and Cabbage behind).  
Farmer returns alone  
Farmer takes Wolf across  
Farmer returns with Goat  
  
\*We now have the Farmer, the Cabbage and the Goat on one side and the Wolf on the other side  
Farmer takes Cabbage across  
Farmer returns alone  
Farmer takes Goat across
3. Notice that  $1+100 = 101$ ,  $2+99 = 101$ ,  $3+98 = 101$ , and so on. There are 50 pairs like this from 1 to 100 that sum to 101, so the answer is  $101 * 50 = 5050$ . **Answer: 5050**
4. There are four big triangles on the top, left, right, and bottom. Then, count the four big triangles that are diagonal. Finally, count the small triangles within each small square. Adding up all of these, we get 16 triangles. **Answer: 16**
5. Let's say that Alan gets an A. Well, from his statement, then Beth would also get an A. But from her statement, Carlos would get an A. And from his statement, Diana would also get an A. So all 4 would get A's, but the problem said only 2 got A's.  
Let's say that Beth gets an A. From her statement, we know that Carlos get an A, and from his statement we know that Diana gets an A. But that makes 3, which is not 2.  
If Carlos gets an A, then Diana gets an A. That makes 2, so  is the right answer. Note that although Beth said "If I get an A, then Carlos will get an A.", that does NOT mean that "If Carlos gets an A, then I will get an A."

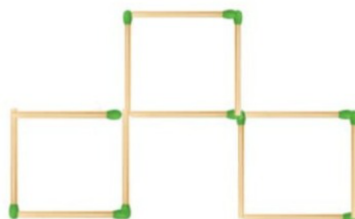
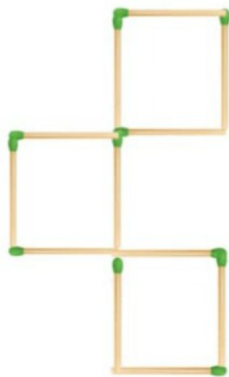
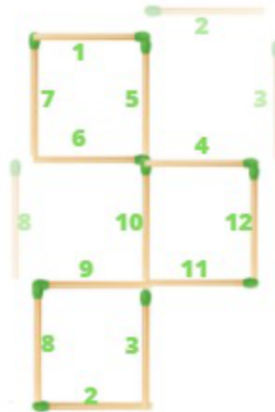
- Notice that  $11 - 9 = 2$ . (The larger number has to end in 1 and the smaller number has to end in 9). Thus, the lowest possible numbers for their house are 19 and 91.

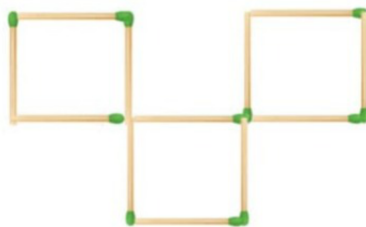
## 5 One Square Less

- The final result should be as shown below:



- Any of these four solutions are valid:





## 6 Which Marble?

The last marble will be **blue**.

Since balls can only be taken out in pairs and you started off with an odd number of blues, there is always going to be one blue left over that you'll keep putting back in the box until it's left on it's own.

## 7 HAPPY

Following the rules of multiplication, we see that 9 times the units digit of the three digit number ends in 7, which means that the digit must be a 3. Carrying out the multiplication, we see that the last two digits of the second product are 17, which means that the hundreds digit in the first product must be a 4. We now have "HA66Y".

The only digit that would work as the units digit of the number starting with 9 is 7. Therefore we have "HA661".

The only multiple of 7 that is two digits and is 7 times a digit is 14. Therefore we have "20661".

## 8 More Riddles

- Let the three digits are 100s digit  $a$ , 10s digit  $b$  and 1s digit  $c$ . Accordingly, the number will be  $100a + 10b + 1c$ . Let the third digit  $c$  be  $x$ .  
The second digit is four times as big as the third digit, thus the 2nd digit  $b = 4x$ . The first digit is three less than the second digit, thus 1st digit  $a = (4x - 3)$ .  
Now there are the following possibilities: since  $x$  is a digit, it must lie between single digit of 1 and 9.  $x$  cannot be 0 or more than 3 because if  $x$  is considered 0, then the digit  $a$  will be 0 and the resulting number will be only two digit number.  $x$  cannot be 3 because if  $x = 3$ , then second digit  $b$  which is  $4x$ , will not be between 1 and 9. So,  $x$  can only take the value of  $c$  as 1 and 2.  
If  $x = 1$  then the third digit  $c$  will be 1, 2nd digit  $b$  will be  $4x = 4$ , and first digit  $a$  will be  $4x - 3 = 4 - 3 = 1$ . Therefore, the three digit number will be **141**.
- $444 + 44 + 4 + 4 + 4 = 500$ .
- Take the first scenario. The first person shakes hands with everyone else, meaning 6 handshakes have happened. Then, the second person shakes hands with everyone else *except* the first person because they have already shaken hands, so that makes 5 handshakes. If you go on like this, the total number of handshakes comes out to be  $6+5+4+3+2+1 = 21$ . **Answer: 21 handshakes.**