1 Introduction

Welcome to Pleasanton Math Circle! We are so excited to work with all of you to solve fun, challenging, and thought-provoking math problems together. This week, we are working on combinations and permutations.

What are combinations and permutations? Combinations and permutations are concepts used to count the number of ways to arrange or select objects. More specifically, in a combination, the order of the objects does not matter, and in a permutation, the order of the objects does matter.

An example of a permutation is a code to the lock, let's say 589. In this case, we do care about the order of the 3 numbers since 598 or 895 will not work. There is only one way to open the lock.

Here is an example of a combination: Let's say you go to a restaurant to pick 3 out of 10 toppings for a pizza, for example, cheese, pepperoni, and sausage. It does not matter what order the ingredients are in; they could also be pepperoni, sausage, and cheese, or sausage, cheese, and pepperoni. It's still the same pizza.

2 Warm-Up

Try the warm-up and ask a teacher if you need help. There are three fruits, an apple, a banana, and a cherry. How many ways are there to arrange them in a line?

For this simple problem, the most straightforward way to do this is probably to list out all the possibilities. Let's let A represent the apple, B represent the banana, and C represent the cherry. Listing them out we have 6 ways:

A	B	C
A	C	B
B	A	C
B	C	A
C	A	B
C	B	A

Note that this is a very slow and inefficient way. Instead, we can consider this: To pick the first fruit, we have 3 options(A, B, or C). Then for the second fruit, we have 2 options left. Lastly, we have 1 option left for the third fruit. This gives the equation $3 \times 2 \times 1 = 6$ ways.

3 Permutations

1. Jack is hungry after arranging his books so Jill decides to get him Animal Crackers as a snack. He eats all but one of each type of cracker and decides to arrange his remaining crackers into an animal parade. If he has nine crackers left, how many different parades can he create?
2. Jack and Jill have a six digit combination lock. The pair decide to hold a competition to see who can figure out the number of total possible combinations first. Jack immediately gets to work and starts listing them all out. Knowing her brother's strong work ethic, a worried Jill asks you for help. How many total combinations are there?
3. Jack has 10 different books; 4 for math, 3 for biology, 1 for history, and 2 for English. If he wants to keep all the biology books together, how many ways are there to arrange all the books on a shelf?

The general equation of a permutation problem, n permute r is:

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

4 Combinations

1. There are 4 balls of different colors in a bin: red, yellow, green, and blue. How many ways can Bob pick 2 balls?
2. Your class has to form a student council of 6 people. If it is required that there are 2 girls and 2 boys and there are 13 boys and 12 girls in your class, how many ways are there to form the council?
3. Champ the dog digs around in a ball pit with 20 balls, each a different color. What is the probability that he picks a pink ball and a yellow ball?

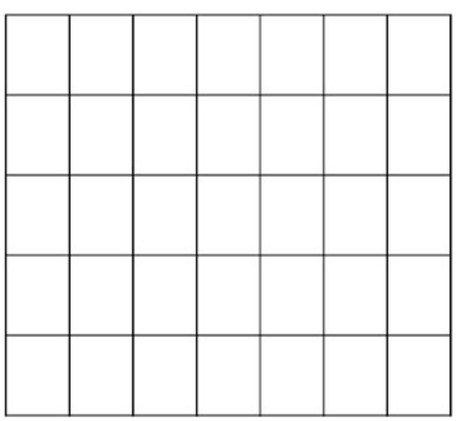
The general equation of a combination problem, n choose r is:

$$ C_{(n,r)} = \frac{n!}{r!\,(n-r)!} $$

Note: Do not simply memorize these formulas, try to understand them

5 Challenge Problems

1. Freddy the frog wants to jump from the point (0,0) to the point (7,5) where there is a tasty fly. If he starts at (0,0) and can only jump to the right and up, how many ways are there to get to his snack?



2. How many different triangles can be formed by connecting the vertices of a regular octagon? Two triangles are different if there is at least one different vertex between them.

3. Amador is the best highschool in Pleasanton. Using the letters in Amador, how many possible words of any length can be formed? Letters cannot be used more times than they appear. For example, "mam" would not be a valid word because "m" only appears once, while "ama" works.

4. Bob, an avid gardener, wants to plant four oak trees, three spruce trees, and five pine trees in a row. How many ways can he do so such that no two pine trees are adjacent?