# 1 Introduction

Welcome to the last Pleasanton Math Circle meeting of the year! We've had so much fun learning with you guys, and we are hoping that next year, we can hold PMC in person. Today we will be exploring geometry. Geometry is all around us, and appears in many different forms. At its core, geometry takes basic ideas like points, lines, angles, and lengths, and finds many ways to relate them to each other. For today, we will mainly focus on learning about shapes, area, and perimeter.

# 2 Warmup/Essential Vocab

Feel free to jump around and answer what questions we know. You are not expected to know all of this.

1. A polygon is equilateral if all of its ____________________ are equal.
2. A polygon is equiangular if all of its ____________________ are equal.
3. A polygon is regular if it is ____________________ and ____________________.
4. A triangle with two sides being equal is called ____________________.
5. A triangle with no sides being equal is called ____________________.
6. A quadrilateral is a polygon with ____________________ sides.
7. A rectangle is a(n) ____________________ quadrilateral.
8. A rhombus is a(n) ____________________ quadrilateral.
9. A square is a(n) ____________________ quadrilateral.
10. An acute angle is ____________________ 90 degrees.
11. A right angle is ____________________ 90 degrees.
12. An obtuse angle is ____________________ 90 degrees.
13. Two angles are complementary if they add up to ____________________ degrees.
14. Two angles are supplementary if they add up to ____________________ degrees.
15. A triangle with all three angles less than 90 degrees is a(n) ____________________ triangle.
16. A triangle with a 90-degree angle is a(n) ____________________ triangle.
17. A triangle with an angle more than 90 degrees is a(n) ____________________ triangle.
18. Two lines are ____________________ if they never intersect each other. Remember that lines extend infinitely.
19. Two lines are ____________________ if they intersect at a right angle.

# 3 Definitions

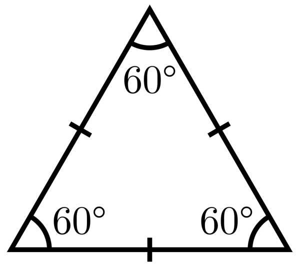
Below are some important definitions that are central to geometry:

**Congruence** is used to describe two things that are the same. This can be two line segments, each with the same length, two angles, each with the same angle measure, or two polygons, each with the same shape and size.

**Similarity** on the other hand only pertains to polygons, and it occurs when two polygons have the same shape, but different sizes. One important use of similarity is similar triangles, which help solve a variety of problems.

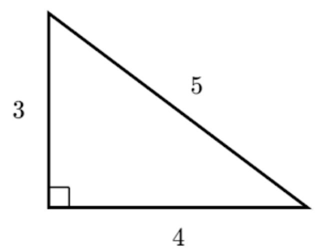
The **perimeter** of a polygon is the sum of its side lengths.

A **regular polygon** is a polygon with n congruent sides and n congruent angles. An example of this is an equilateral triangle, which has 3 congruent sides, and 3 angles each with a measure of 60°. An equilateral triangle is shown below:



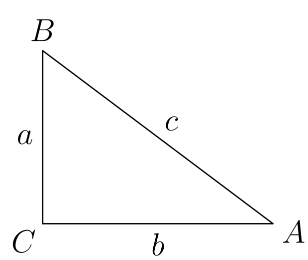

A **right triangle** is a triangle that has an angle of 90°. An example of such a triangle is shown below, in the next section.

# 4 Triangles




In a right triangle, the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides. This is known as the **Pythagorean Theorem**. One of the most common right triangles is shown above.

## 4.1 Pythagorean Theorem




The Pythagorean Theorem says that if one these statements is true, then the other is true as well.

- $C$ is a right angle
- $a^2 + b^2 = c^2$

The area of a triangle is usually found by using the formula $A = b * h/2$, where A is the area, b is the length of the base, and h is the length of the altitude to the base.

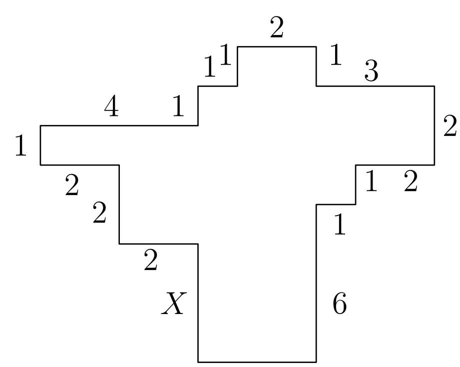
If a triangle is isosceles, then the two angles opposite the congruent sides are also congruent.

## 5 Conceptual Questions

1. True or false: all equilateral triangles are equiangular. ____________________
2. True or false: all equiangular triangles are equilateral. ____________________
3. True or false: all equilateral polygons are equiangular. ____________________
4. True or false: all equiangular polygons are equilateral. ____________________
5. The sum of a triangle's interior angles always equals ____________________ degrees.
6. The sum of a quadrilateral's interior angles always equals ____________________ degrees.
7. The sum of a pentagon's interior angles always equals ____________________ degrees.
8. The sum of the angles of a polygon with $n$ sides is given by the expression ____________________.
9. An equilateral triangle has three ____________________-degree angles.
10. Say we are given two shapes: shape A and shape B. Shape B is an enlarged version of A; the shape is the same, but each side length of B is twice as big as the corresponding length of A. The area of B is ____________________ times as much as the area of A.
11. If in the previous problem each side length in B is $n$ times the corresponding length in A, the area of B is ____________________ times the area of A.

## 6 Interesting Problems

1. (2012 AMC 8 #5) In the diagram, all angles are right angles and the lengths of the sides are given in centimeters. Note the diagram is not drawn to scale. What is the length in $X$, in centimeters?




**(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

2. Which of the following sets of side lengths will form a right triangle?
**(A)** $\{\frac{1}{3}, \frac{1}{4}, \frac{1}{5}\}$ **(B)** $\{\sqrt{3}, \sqrt{4}, \sqrt{5}\}$ **(C)** $\{9, 16, 25\}$ **(D)** $\{13, 14, 15\}$ **(E)** $\{30, 40, 50\}$

3. (1985 AJHSME #12) A square and a triangle have equal perimeters. The lengths of the three sides of the triangle are 6.2 cm, 8.3 cm and 9.5 cm. The area of the square is

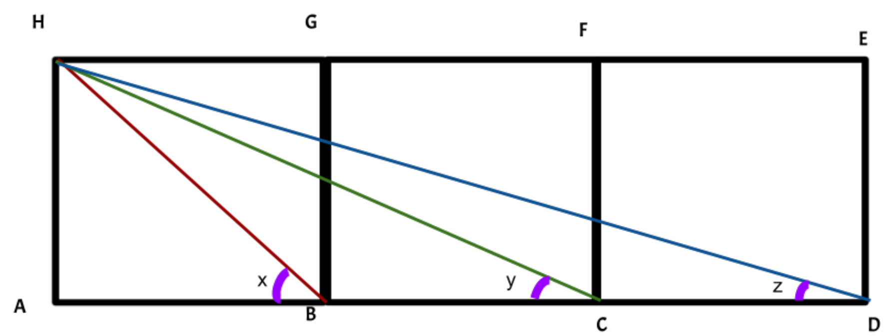
**(A)** 24 $\text{cm}^2$ **(B)** 36 $\text{cm}^2$ **(C)** 48 $\text{cm}^2$ **(D)** 64 $\text{cm}^2$ **(E)** 144 $\text{cm}^2$

4. (2012 AMC 8 #23) An equilateral triangle and a regular hexagon have equal perimeters. If the area of the triangle is 4, what is the area of the hexagon? (Note: this is an excellent problem to think creatively with. There's a trick you can use that will give you the answer in around 15 seconds)

   **(A)** 4 **(B)** 5 **(C)** 6 **(D)** $4\sqrt{3}$ **(E)** $6\sqrt{3}$

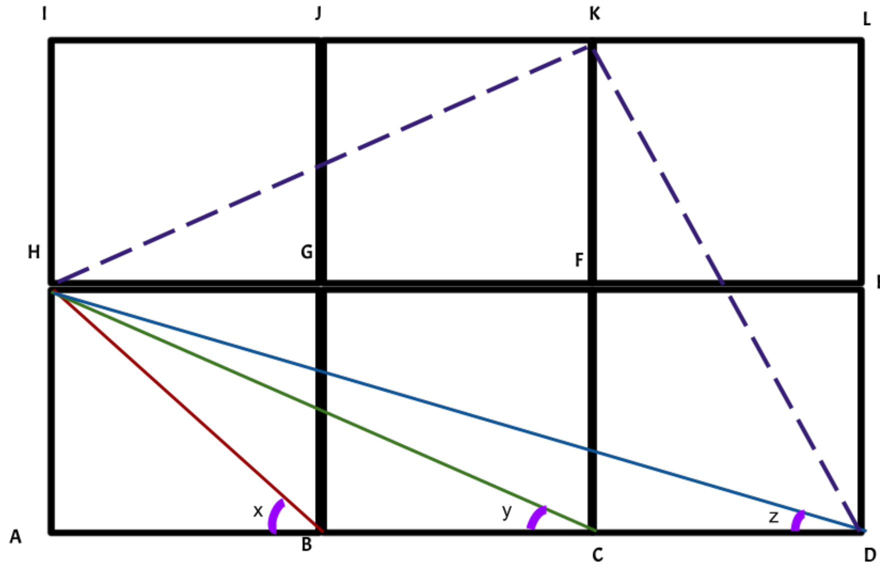
# 7 Three triangles and Three squares

We can find the angles of a triangle using squares! Don't believe me? Try it yourself: Find the sum of the angles x, y, and z.




Hints:

- Triangles are inside squares, so we know the value of angle x, what is it?
- You don't need to know all the values of x, y, and z to find the sum.
- Add 3 square directly above the original 3 squares and label the new points. Then draw a triangle from points H, K, and D. This is all done in the image below




- Do you see any similarities in the triangles we created? How are triangle CAH and KLD similar?