1 Introduction

Welcome back to the Pleasanton Math Circle (PMC)! Today we will be exploring series, sequences, and pascal's triangle. Feel free to ask a teacher if you need some help. We will be going over some of these questions throughout the class as well.

2 Warm Up

First what is a sequence? A sequence is a string of numbers that follow a certain pattern. For example the numbers 1, 2, 3, 4, 5... are in a sequence because they follow the pattern of adding one to the previous number.

Find the pattern for the sequence:

- 0, 2, 4, 6, 8...
- 1, 3, 5, 7, 11, 13...
- 1, 3, 6, 10, 15
- 3, 9, 27, 81, 243, 729...
- 1, 121, 12321, 1234321, 123454321, 12345654321...
- 18, 12, 19, 11, 20...

3 Sequences

3.1 Arithmetic

Arithmetic sequence starts with a number a_1. To generate the next terms, you add a constant number repeatedly.

Let's look at an example. You want to buy several bottles of soda, one bottle costs 1 dollar, but any bottle you buy after the first costs two dollars. We can write out the prices for buying 1, 2, or 3 bottles and so on of soda:

$$1, 3, 5, 7, 9, ...$$

You start with one dollar and add two every time afterwards. Consequently, the "generating formula" for this sequence is 2n 1. Plugging in different values for n will generate the nth term in the sequence.

Identify the following:

1. Consider the sequence:

   $$45, 50, 55, 60, ...$$

   What is the formula for this?

2. You want to buy ten packages of cheese, and you know that first one costs 5 dollars. Every other one costs 6 dollars. What is the total price of the ten packages?

3. On January 1st, Bubba eats 2 chocolate. On January 2nd, Bubba eats 5 chocolates. On January 3rd, Bubba eats 8 chocolates. On every day of January, Bubba eats 3 more chocolates than the day before. How many chocolates does Bubba eat in January?

3.2 Geometric

Geometric sequence starts with a number a_1. To generate the next terms, you multiply by a constant number repeatedly.

Imagine you have two friends at your party, and each of them may invite two friends as well, and so on and so forth. You start with two friends, than have four more friends, then eight more friends, etc. The number of new friends coming to your party written out are:

$$2, 4, 8, 16, ...$$

The formula is 2^n.

1. What is the formula for:
$$14, 70, 350, 1750, ...$$
2. Cookie Monster has a full cookie. Every time he sneezes, he eats half of however much cookie is in front of him. For example, if he sneezes once, he will eat half of his cookie. If he sneezes again, he will eat a fourth of his cookie. How many times does he sneeze if he looks down and sees $\frac{1}{16}$th of a cookie?
3. How much cookie does Cookie Monster eat after he sneezes 5 times? How much cookie does he eat after sneezing 10 times? How much cookie does he eat after sneezing an infinite number of times?

3.3 Figurate

Figurate sequence is the one that demonstrates the amount of points needed to draw some geometric figures.

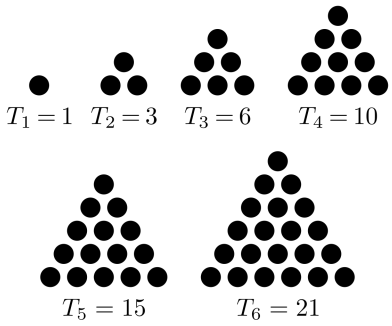
$$1, 3, 6, 10, ...$$

is a triangular sequence, shown on the picture below.

$$1, 4, 9, 16, ...$$

is a square sequence.What is a formula for the square sequence?

1. What is the 7th triangular number? What is the 10th? Is there a fast way to do this?
2. What is the 10th hexagonal number?



4 Fibonacci Fun

It was introduced to the West by an Italian mathematician Leonardo Fibonacci, who wrote of his findings in the book Liber Abaci. It is a series of numbers; each term is found by adding the two numbers before it, starting with 1 and 1, going up from there. Fill out the first 10 terms of the Fibonacci sequence.

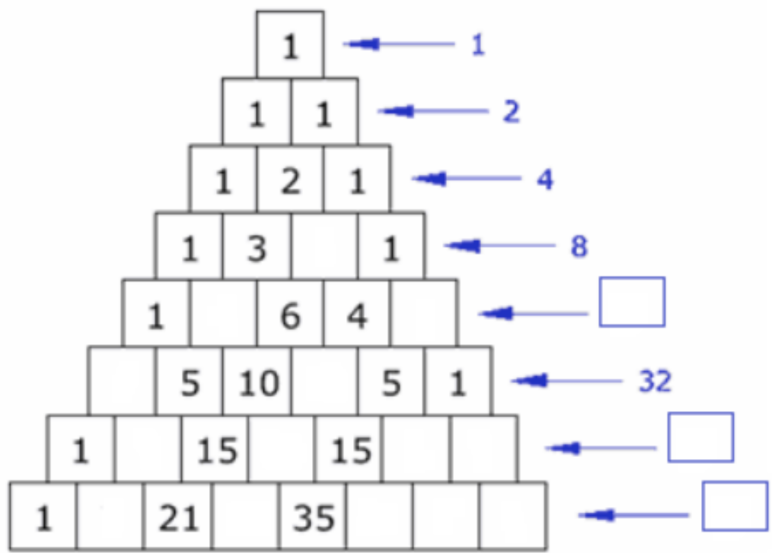
n	0	1	2	3	4	5	6	7
F_n	1	1	2	3	5			

Fibonacci can also be written as $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 2$

What if instead of 1 and 1, we start with 1 and 2? This will generate the Lucas numbers.

5 Pascal's Triangle

Notice a pattern in the image below, and fill in the rest of the empty squares. On the right is the sum of the row.



You just wrote out Pascal's triangle! You can keep adding more rows to Pascal's triangle, but lets stop here for now.

1. What are some is the patterns you see (there are many!)?
2. Find the Fibonacci sequence in Pascal's triangle.

6 Super Ultra-Fun Arithmetic Sequence Problem

An arithmetic sequence is a sequence in which each term after the first is obtained by adding a constant to the previous term. Each row and each column in this 5×5 array is an arithmetic sequence with five terms. What is the value of X (AMC 8)?

1				25
		X		
17				81