1 Introduction

Triangulation is an important process where you use relationships between triangles to find the relative distances between points and positions of certain places. This is used in important things like GPS (Global Positioning System), earthquake seismology (finding out where the epicenter of the earthquake will be), and more. Today, we will be exploring how triangulation works through puzzles and games and performing some basic triangulation calculations!

2 Delaunay Triangulation and Voronoi Diagrams

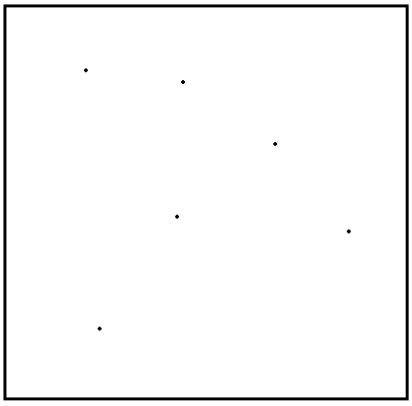
Suppose you have 6 friends who have just discovered a new land. Although they are friends, they each have their own ideas of what they want to do with the land, so they decide to split it up fairly. However, each friend also wants a certain location (as shown by the dots below) and the land that is closest to their location. The friends are stumped on how to do this accurately, so they ask for your help.

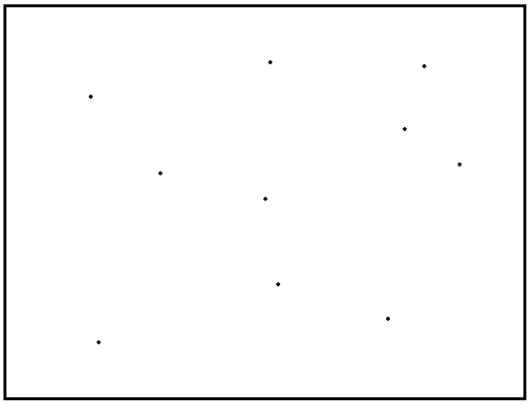
Can you think of an accurate process by which you can split the land based on the friends' requirements?

Seems kinda hard right? I mean you *could* just guess and check and you'll eventually split the land properly, but you don't have that much time to waste! Another group of friends just asked you to help them with the same thing, and their group has 20 people! So, you ask your wonderful PMC lecturers and they have come up with a process:

1. First mark the locations of each friends' special spot (already done for you).
2. Now you have to create triangles between the points. However, there is a special rule. For every triangle that you make, you must also draw the circle that contains all three vertices of the triangle. No circle can contain inside it any of the other locations. It is very important that you follow these rules or else this process will not work.
3. Now that you have all the correct triangles and all of the corresponding circles, mark the center of each circle with a dot.
4. Now, remember the request of the friends: each piece of land can only have the land that is closest to the special location. The dots that you have just made are the intersection points of the pieces of land. Now all that you have to do is join the points in such a way that follows the requirements!

Now that you know how this process works, try it again for the group of 10 friends who have the same problem and the same requirements.





This process is called triangulation. When you drew all the correct triangles, you got the Delaunay Triangulation, and once you created all the cells, you made the Voronoi Diagram! This is really useful for all different sciences such as biology (with cells), chemistry (with atoms), and physics (with nuclear particles).

3 Challenge!

Now try to make the Voronoi Diagram for these 20 locations: