1 Warm-Up

Try this: List the first ten prime numbers.
Answer: Remember, 0 and 1 are not considered prime numbers. Thus, the first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

Try this: List the first ten composite numbers.
Answer: Remember, 0 and 1 are not considered composite numbers either. Thus, the first ten composite numbers are 4, 6, 8, 9, 10, 12, 14, 15, 16, and 18.

2 Sieve of Eratosthenes

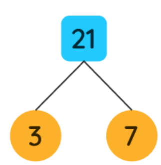
The image below shows what the completed table up to 100 should look like once you're done.

1	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

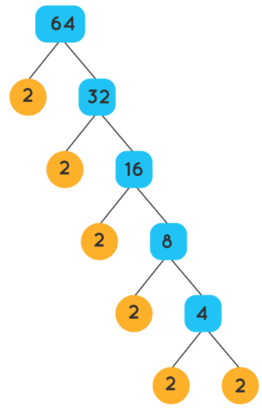
3 Prime Factorization

Try it! Find the prime factorization of each of the following numbers:

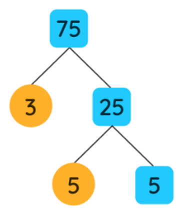
1. 21:



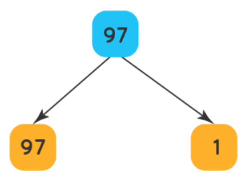
2. 64:



3. 75:

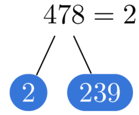


4. 97:



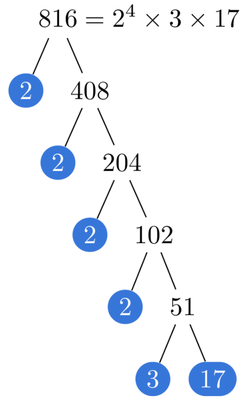
5. 478:

$$478 = 2 \times 239$$



6. 816:

$$816 = 2^4 \times 3 \times 17$$



4 GCD

1. **Find the GCD of 36 and 75.**
   $36 = 2 \cdot 2 \cdot 3 \cdot 3$
   $75 = 3 \cdot 5 \cdot 5$
   The only common factor between the two is 3, so the GCD of 36 and 75 is 3.

2. **Find the GCD of 105 and 84.**
   $105 = 3 \cdot 5 \cdot 7$
   $84 = 2 \cdot 2 \cdot 3 \cdot 7$
   The common factors between the two are 3 and 7. The GCD of 105 and 84 is thus 21.

5 LCM

1. **Find the LCM of 7 and 15.**
   $7 = 1 \cdot 7$
   $15 = 3 \cdot 5$
   Since they each have different factors, the LCM is $3 \cdot 5 \cdot 7 = 105$.

2. **Find the LCM of 8 and 14.**
   $8 = 2 \cdot 2 \cdot 2$
   $14 = 2 \cdot 7$
   2 appears the greatest number of times in 8 (3 times). 7 appears one time in 14. Therefore, the LCM of 8 and 14 is $2 \cdot 2 \cdot 2 \cdot 7 = 56$.

6 Interesting Problems

1. *Answer*: Since the two prime numbers sum to an odd number, one of them must be even. The only even prime number is 2. The other prime number is $85 - 2 = 83$, and the product of these two numbers is $83 \cdot 2 = \boxed{\textbf{(E) } 166}$.

2. *Answer*: The prime numbers between 10 and 20 are 11, 13, 17, and 19. The sum of these numbers is 60. Next, to find the prime factors of 60, we must do prime factorization. We find that the prime divisors of $60 = 2 \cdot 2 \cdot 3 \cdot 5$. The greatest number out of these prime divisors is 5, so the answer is $\boxed{\mathbf{5}}$.

3. *Answer*: Notice that 44 and 38 are both even, while 59 is odd. If any odd prime is added to 59, an even number will be obtained. However, the only way to obtain this even number(common sum) would be to add another even number to 44, and a different one to 38. Since there is only one even prime (2), the middle card's hidden number cannot be an odd prime, and so must be even. Therefore, the middle card's hidden number must be 2, so the constant sum is $59 + 2 = 61$. Thus, the first card's hidden number is $61 - 44 = 17$, and the last card's hidden number is $61 - 38 = 23$.

   Since the sum of the hidden primes is $2 + 17 + 23 = 42$, the average of the primes is $\frac{42}{3} = \boxed{\textbf{(B)}14}$.

4. *Answer*: We wish to find possible values of a, b, and c. By finding the greatest common factor of 12 and 15, we can find that b is 3. Moving on to a and c, in order to minimize them, we wish to find the least such that the least common multiple of a and 3 is 12, $\rightarrow$ 4. Similarly, with 3 and c, we obtain 5. The least common multiple of 4 and 5 is 20 $\rightarrow \boxed{\textbf{(A)}20}$.

5. *Answer*: Since the question asks which of the following will never be a prime number when p is a prime number, a way to find the answer is by trying to find a value for p such that the statement above won't be true.

   A) $p^2 + 16$ isn't true when $p = 5$ because $25 + 16 = 41$, which is prime

   B) $p^2 + 24$ isn't true when $p = 7$ because $49 + 24 = 73$, which is prime

C) $p^2 + 26$

D) $p^2 + 46$ isn't true when $p = 5$ because $25 + 46 = 71$, which is prime

E) $p^2 + 96$ isn't true when $p = 19$ because $361 + 96 = 457$, which is prime

Therefore, $\boxed{\text{C}}$ is the correct answer.

6. *Answer*: To find the LCM of two numbers, we must prime factorize both of them. Finding the prime factors of 14 and 8, we get:
$14 = 2 \cdot 7$
$8 = 2 \cdot 2 \cdot 2$
The LCM we get is 56. However, this isn't a perfect square. With a little bit of experimentation, we can find that 784, which is a perfect square, is a multiple of 8. Then, we can divide 784 by 14 and find out it is also a multiple of 14. Therefore, $\boxed{784}$ is the correct answer.