1 Introduction

Welcome to the second Pleasanton Math Circle Meeting (PMC)! Today we will be exploring some arithmetic and geometrical tricks. Feel free to ask a teacher if you need some help. We will be going over some of these questions throughout the class as well.

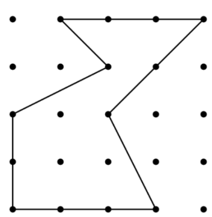
2 Warm Up

Try to solve the equation below without actually multiplying all the numbers out.

$$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)...\left(1+\frac{1}{9}\right)=? \tag{1}$$

3 Area

Find the area and perimeter of the shape below. What are some different ways you can do this?

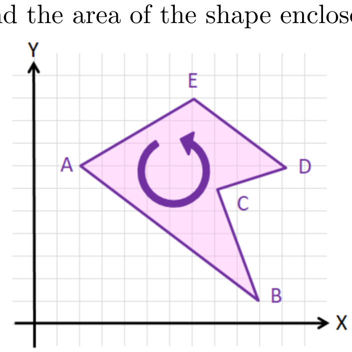


4 Shoelace Theorem

How did you find the area of the shape above? Did you count the squares and the partial squares and guesstimate? With shoelace theorem, you can easily find the exact area of anything with coordinates!
Here are the steps:

1. List all the vertices in anticlockwise order. (e.g., A,B,C,D,E) in a table, and note the x and y coordinates in two separate columns of the table.
2. Calculate the sum of multiplying each x coordinate with the y coordinate in the row below (wrapping around back to the first line when you reach the bottom of the table). The lines should crisscross like shoelaces.
3. Calculate the sum of multiplying each y coordinate with the x coordinate in the row below (wrapping around back to the first line when you reach the bottom of the table).
4. Subtract one sum from the other, divide the difference by two, and make the number positive (absolute value). You have the area of the polygon!

Find the area of the shape enclosed by (2,7), (10,1), (8,6), (11,7), and (7,10).

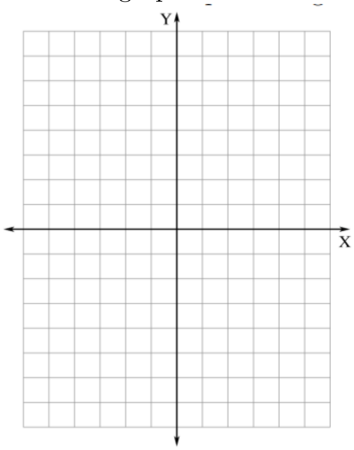


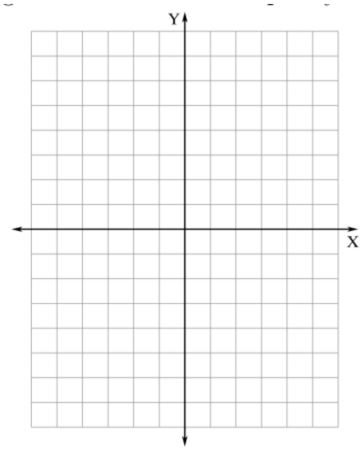
Vertices	x	y
A	2	7
B	10	1
C	8	6
D	11	7
E	7	10

Now, using the steps listed above, find the area of the polygon with vertices at the following points: (3, -3), (-5, 1), (2.3), (5, 5), (-2,5), (1,-4).

5 Taxis and Crows

Imagine you're driving a taxi in a big city. This perfect city is made out of many many streets stretching out both North-South and East-West at perfect right angles. This should look pretty familiar. It looks just like a normal graph!

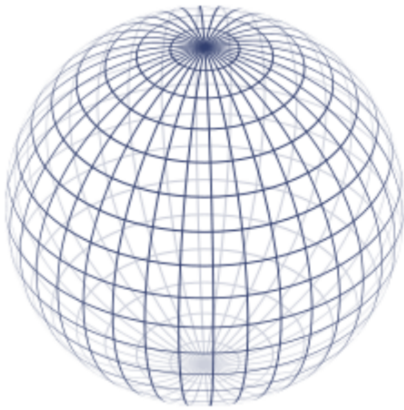


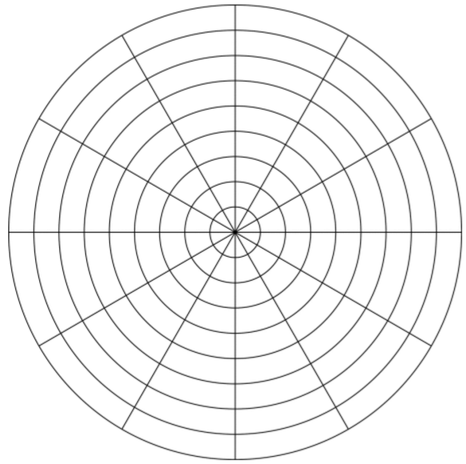


1. Let's say you are driving a customer from the origin (0,0) to point (3,5)? What is the taxi distance? How many different paths of shortest length can you take? Remember, it's as the taxi drives, not as the crow flies.
2. A crow wants to know all the places it can go that are four units away. Try drawing all the points the crow can go on the left graph.
3. Let's do the same for the taxi. What are all the places it can travel with enough gas to drive 4 units?
4. If π can be calculated in the crow's map as circumference divided by diameter, what is the "π" of a taxi?

6 Spherical Geometry

Now you're an astronaut! And the earth is round. Like this:





1. Can you make two parallel lines?

2. Make a triangle on both maps. How do you even make a triangle on a sphere?

3. How do you calculate the side lengths?

7 Complementary Counting

How many two-digit numbers are not divisible by 3? Instead of counting 10, 11, 13, 14 and so on, let's try a new technique: complementary counting.

1. How many two-digit numbers are there?

2. How many two-digit numbers are divisible by 3? (Hint: Find the smallest and largest two-digit number divisible by 3).

3. So, how many two-digit numbers are NOT divisible by 3?

Based on this example, what is complementary counting? Instead of counting the numbers we do want, we did something much easier – the exact opposite.

Here are the steps we took:
1. First, we counted the total possibilities (all two-digit numbers).
2. Next, we counted the number of possibilities that don't work (two-digit numbers that are divisible by 3).
3. Finally, by subtracting the number of possibilities that don't work from the total number of possibilities, we found the number of possibilities that do work (two-digit numbers that are not divisible by 3).
To summarize: Possibilities that DO work = TOTAL possibilities - Possibilities that DON'T WORK

8 Factorials

When you take the factorial of a number, you multiply the number by every natural number below it. (Remember: 0 is not a natural number!) Factorials are represented by an exclamation mark (!). So, for example, $3! = (3)(2)(1)$.

1. What do $5!$ and $6!$ equal?

2. Try to write the general factorial of any positive number, n.

3. Santa has 4 reindeer and he wants 3 of them to fly his sleigh. He always makes his reindeer fly in a single-file line. How many different ways can he arrange his reindeer?