1 Introduction

In mathematics, drawing pictures often helps especially when we are given a large set of values and asked to compare. **Venn diagrams** allow us to untangle and visualize data, making common applications like addition and subtraction easier.

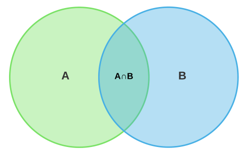

Figure 1: Venn Diagram

Label the diagram with "**A**", "**B**", and "**A and B**" as shown.

2 Warm-up

In a classroom with 10 students, everyone plays at least one sport after school. The school offers the option of volleyball and basketball. 7 students play volleyball and 3 play both sports. How many students play basketball? (*Hint: Think of which piece of given information is the easiest to start with.*)

Label the left circle "basketball" and the right circle "volleyball" so the overlap represents both sports. Since we know that 3 play both, write 3 in the middle. 7 players play volleyball INCLUDING the 3 in the overlap, so there are 7 - 3 or 4 players that play only volleyball. This gives up 4 + 3 players accounted for which means that 10 - 7 players play only basketball. Add 3 to represent the students who also play volleyball to get 6 students who play basketball.

To solve, first draw a Venn diagram and label it. Place the known data into the corresponding sections. Now, fill in the missing value given the total.

3 Try It Yourself

1. A school took a survey to see students' preferred transportation methods to and from school. The statistic showed that 22 students walk, 17 bike, and 12 use both. How many students did they survey in total?
   To get the total, we add the kids who only walked and the kids who biked in general (because that represents both the bikers and the people who are ok with either). So, 22 total walkers - 12 (both) is 10 only walkers. Add the 17 total bikers to get 27 students surveyed in total.

2. A student was curious to see what her peers did over winter break. She asked her 30 classmates and found that 19 of them vacationed in Lake Tahoe and 17 went to Disneyland, and 8 lucky kids did both.

   Add up all the values in your diagram. Is it the same as the total? If not, why did you fall short?

   No. The students who stayed home during break aren't represented in the diagram.

   In Venn Diagrams, we must look at A, B, both but also consider **neither A nor B**. This number is denoted outside the Venn diagram and should be accounted for in the total.
   How many students did only one activity over break? How many did none? *The kids who only did one activity either went to Lake Tahoe or Disneyland. The kids who only went to Lake Tahoe is 19 - 8 (the kids who did both) or 11 while the kids who only went to Disneyland is 17 - 8 or 9. So the total*

kids who did one activity is 9 + 11 = 20. Since 20 kids did 1 activity and 8 kids did 2 and there are a total of 30 kids, that means that the remainder are the kids who stayed home. So, 30 - (20 + 8) = 2.

3. In genetics, there are dominant and recessive alleles. The alleles come in pairs, one from the mother and the other from the father. Let's denote the dominant allele for big ears as B and the recessive as b. BB and Bb result in big ears while one must receive both recessive alleles (bb) for small ears. If the probability of getting one recessive allele is .5 and the probability of receiving at two dominant genes is .25. What chance does Dumbo have of getting small ears?
Draw a Venn Diagram and label the left circle BB, right circle bb, and the overlap Bb. Since the probability of getting one recessive allele is .5, that means the overlap is .5. We know that the probability of getting a dominant allele in general is .75, so that means that getting BB is .75 - .5 = .25. Knowing that total probability is 1, we subtract .25 and .5 from 1 to get .25.

4 Sum It All Up

Now that you are more comfortable with Venn diagrams, take Problem 1 and try to find the total number of students in terms of values x y and z. Use the diagram below as a guide to find the total value in terms of X, Y, and Z.

Total = (X - Y) + (Z - Y) + Y or simplified to (X - Y) + Z.

5 Challenge

1. Great news! Everyone in Mr. Lomas' math class earned an A+ on the last math test, so he decided to treat them to ice cream. At the shop, there were three flavors to choose from: chocolate, vanilla, and sorbet. They could either get 1, 2, or all flavors (one scoop per flavor). Unfortunately, the register is broken, but Mr. Lomas noted that there were 26 scoops of chocolate in total, 5 people with three scoops, 2 people who chose only sorbet, 17 cups with 2 scoops, 10 people who decided to get chocolate and vanilla, 5 strictly vanilla-lovers, and 4 people who got chocolate and sorbet. How many students were there total?

   Fill out the known information: 5 in the center, 2 in the "only sorbet," and 10 in the overlap between chocolate and vanilla, and 4 in the overlap of chocolate and sorbet. Since there is a total of 17 "two flavors" that means there are 3 vanilla-sorbets and there are a total of 26 chocolate scoops so 26 - 5 (three-flavors) - 14 (two-flavors) = 7 "only chocolate." Add the total of all categories to get 36 students.

   If the shop charges 50 cents per scoop, how much did Mr. Lomas spend?

   Each flavor = 1 scoop, so all the single flavors (total 14) should be x 1, all the double flavor (total 17) x 2, and the 5 triple flavor should be x 3 to get 63 scoops. Each scoop is 50 cents, so multiply 63 x .5 to get 31.5 dollars.

2. You are playing a carnival game where there are red, blue, and purple (both red and blue) bottles, and you have to guess the number of purple bottles. You see 18 bottles in front of you, but they are faced down so you can't see the colors. The carnival worker tells you that 11 have a blue bottom and 13 of them are red, and 3 are clear. Try to win the game using a Venn diagram.
There are 18 bottles total. 3 are clear (none) so we know that there are 15 bottles that are blue, red, or both. Call the purple bottles x so that (blue bottles - x) + red bottles = 15. You know that there are 13 red bottles so (blue bottles - x) has to be 2. Since there are 11 blue bottles total and that represents (only blue = blue bottles - x) only blue + x, substitute in 2 to get x = 9.

3. Referring to the formula you created, try writing Y in terms of X and Z and the total.
Rearrange the formula total = (X - Y) + Z to get Y = X + Z - total. Applying this to previous problem, x = 13 + 11 - 15 = 9.