1 Introduction

Today, we will be looking at Halloween math. If you can solve some of the tougher problems, you'll get candy!

2 Warm-up: Numbers in Trick-or-Treating

1. Chris and Gaurav are trading candies. They agree to the following trades:

   - **1** Twix bar = **4** Kit Kat bars
   - **3** Skittle bags = **10** Swedish Fish bags
   - **4** Kit Kat bars = **9** Skittle bags
   - **1** Lollipop = **3** Skittles bags

   If they agree to the trade "1 Twix bar = 4 Kit Kat bars" they would also agree to the trade "2 Twix bar = 8 Kit Kat bars" or "3 Twix bar = 12 Kit Kat bars." Assuming that Chris and Gaurav have enough candy for any proposed trade:

   (a) How many **Skittles** bags can Chris get from 10 **Lollipops**?

   (b) How many **Swedish Fish** bags can Chris get from 3 **Lollipops**

   (c) How many **Swedish Fish** bags can Chris get from 1 **Twix** bar?

2. Neel is going trick-or-treating and arrives at Sri's front door. Sri gives out candy in an interesting way:

   Hat

   - He gives **2** candies to those wearing red hats
   - He gives **1** candies to those wearing blue hats
   - He gives **0** candies to those wearing no hats

   Costume

   - He will give an **additional 5** candies (in addition to above) to those wearing funny costumes
   - He will **triple** the number of candies he gives (from above) to those wearing sad costumes

   Shoes

   - He will **double** the number of candies he gives from the person's hat and costume to those wearing black shoes
   - He will give an **additional 4** candies he gives (from above) to those who are not wearing which shoes.
   - He will give an **additional 2** candies he gives (from above) to those who are not wearing shoes at all.

   Neel can choose to wear a red, blue, or no hat. He can choose to wear a funny or sad costume. Finally, he can choose to wear black, white, or no shoes. What is the greatest possible number of candies that Neel can receive from Sri (and what costume should he wear)? What is the least possible number of candies that Neel can receive (and what costume would he wear for this)?

3 Candy Piles

Alex, Brian, and Craig all go trick-or-treating to the same houses and all come home with the same number of pieces of candy. Each of the following questions are different scenarios for the amount of candy they have.

1. Alex creates groups of 3 with his candy, and after he can make as many groups of 3 as he can, he sees that he has 2 pieces left over. Brian creates groups of 2 with his candy in this same way, and after he can make as many groups of 2 as he can, he sees that he has 1 pieces left over. Given only this information, what is the smallest possible number pieces of candy that Alex and Brian could each have?

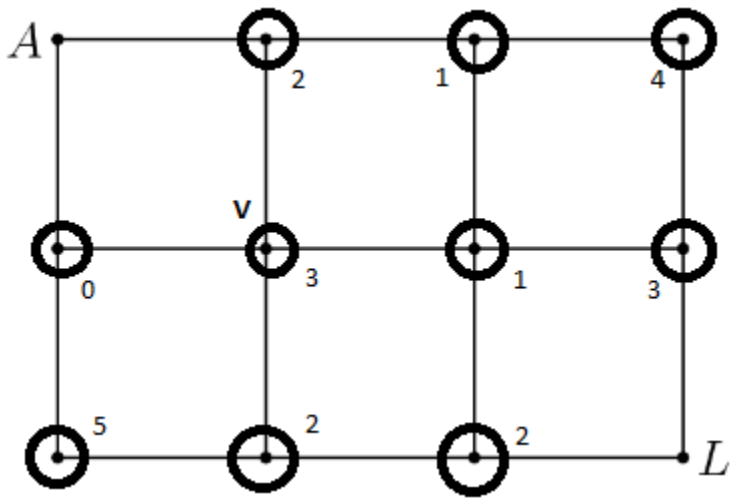
2. Alex creates groups of 5 with his candy and sees that he has 3 candies left over. Brian creates groups of 4 with his candies and sees that he has 2 candies left over. Given this information, what is the smallest possible number of pieces of candy that Alex and Brian could each have?

3. Alex creates groups of 7 with his candy and sees that he has 1 candy left over. Brian creates groups of 5 and sees that he has 1 candy left over. Given that **Alex and Brian each have more than 80 candies**, what is the smallest possible number of pieces of candy that Alex and Brian could each have? What is the second smallest possible number of pieces of candy that Alex and Brian could each have?

4. **Challenge** Alex creates groups of 5 with his candy and sees that he has 4 candies left over. Brian creates groups of 7 with his candy and sees that he has 6 left over. Craig creates groups of 9 with his candy and sees that he has 8 left over. Given only this information, what is the smallest possible number of pieces of candy that Alex, Brian, and Craig could each have?

4 Trick-or-Treat Routing

It is Halloween and the map below is a map of YOUR city. You are currently at point **A**, and you will end your trick-or-treating at point **L** only walking on roads. Each circle around a point represents a house and each edge on the map (there are 17 of these total) represents a road. You can not travel on a road twice, but it is okay to visit a house twice. Each number next to a point displays how many pieces of candy a house will give, but you won't have to worry about this until later.



1. In our first scenario, you are in a rush to get home! What is the least amount of roads you have to walk on to get home?

   (a) Draw such a path that you could take on the map.

   (b) How many such "shortest paths" are there (it takes you the same time to walk any 1 road)?

(c) For any house you pass, you collect the number of candies indicated on the map. Of all possible "shortest paths," what is the greatest possible number of candies you can collect?

(d) The point on the map labeled **V** is a vampire's house. How many "shortest paths" can you take which avoid going to the vampire's house?

2. You are no longer in a rush. There are 17 roads, and you can not take any road twice (you can visit houses twice though.)

   (a) **Challenge** What is the greatest possible number of these 17 roads that we can take to end at point **L**?

   (b) **Challenge** How many such "longest paths" are there? (**Hint** for these two problems: Why can't you use all 17 roads? Using certain roads forces you to not be able to use others. Think about minimizing the number of roads you will not use).

5 Halloween Brain Teasers

1. Mr. Green had less than 20 sweets left at the end of Halloween. When the doorbell rang, he decided he would give all the sweets away. There were 2 trick-or-treaters when he opened the door, but when he divided the candies in two, there was one left over. At that point, he noticed another trick-or-treater behind the intial two. Again, he divided the candies evenly among them, but there was still one left over. Finally, another trick-or-treater came to his door, but when Mr. Green divided his sweets into four, there was still one left over. How many sweets did Mr. Green have left over when the doorbell rang?

2. Suppose:

   - ghost + ghost + ghost = 12
   - ghost + pumpkin + pumpkin = 20
   - pumpkin + (3 bats) + (3 bats) = 14

   So what does pumpkin + ghost + (2 bats) = ?

3. **Challenge:** Alice, Bertha, and Cathy are the winners of a school drawing for a pile of Halloween candy, which they are to divide in a ratio of 3 : 2 : 1, respectively. Due to some confusion they come at different times to claim their prizes, and each assumes she is the first to arrive. If each person takes what she believes to be the correct share of candy, what fraction of the candy goes unclaimed?

4. **Challenge:** What is the probability that Halloween will fall on a Friday or Saturday in a given year? What is the probability that it is on a full moon (which occurs every 29.5 days)?

6 More Spooky Season Problems

1. A witch and a wizard go into a potion shop with a total of $30.00 to spend. Broomsticks cost $4.50 each and wands cost $1.00 each. The witch and the wizard plan to buy as many broomsticks as they can, and use any remaining money to buy wands. Counting both broomsticks and wands, how many items will they buy?

2. On Halloween, Carl ate 1/3 of his candies and then gave 2 candies to his brother. The next day he ate 1/3 of his remaining candies and then gave 4 candies to his sister. On the third day he ate his final 8 candies. How many candies did Carl have at the beginning?

3. Each year, I trick or treat on a rectangular block in my town. One side of the block is 2 times the length of the other side. If the perimeter of the entire block is 564 yards, how long is each side?

4. You just put on your new Halloween costume and you and your friends are going trick or treating. At the first house you get 5 pieces of candy, at the second house, you get 10, and at the third house, you get 15. If the pattern continues and there are 12 houses on your street, how many pieces of candy will you get in all? (find the pattern first)

   Challenge: If you eat 2 pieces of candy between each house, how many would you have after all 12 houses?

   Super Challenge: If you went trick-or-treating with 8 friends and they all got the same amount of candy as you, how many pieces of candy did you and your 8 friends get in total?